AP Calculus AB Practice Exam

CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—25
A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION.

Directions: Solve each of the following problems. After examining the choices, select the choice that best answers the question. No credit will be given for anything written in the test book.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. $\int_{4}^{7} \frac{1}{(x-3)^2} dx =$

   A. $\frac{1}{27}$

   B. $\frac{3}{4}$

   C. $\frac{1}{9}$

   D. $\ln 4 - \ln 3$

   E. $\ln 16 - \ln 9$

2. $\int_{\pi/2}^{x} \cos t\,dt =$

A. $\cos x$
B. $-\cos x$
C. $\sin x - 1$
D. $\sin x$
E. $1 - \sin x$

3. $\int_{1/e}^{e^2} \left(\frac{x^3+1}{x}\right)dx =$

A. $\frac{e^6-1}{6e^2}$
B. $\frac{e^{12}+2e^6-1}{6}$
C. $\frac{e^9+e^3-1}{e^3}$
D. $\frac{e^9+9e^3-1}{3e^3}$
E. $\frac{e^9-1}{e^3}+\ln 2$

4. A particle moves along the y-axis so that its position at time $0 \le t \le 20$ is given by $y(t) = 5t - \frac{t^2}{3}$. At what time does the particle change direction?

A. 5 seconds
B. 7.5 seconds
C. 10 seconds
D. 15 seconds
E. 18 seconds

5. Find the average value of $y = x^3\sqrt{x^4-1}$ on the interval [1,3].

A. $216\sqrt{5}$
B. $\frac{216\sqrt{5}}{3}$
C. $80\sqrt{5}$
D. $\frac{80\sqrt{5}}{3}$
E. $\frac{640\sqrt{5}}{27}$

x	1	3	5
f(x)	4	k	3

6. Given that f is a continuous function on the interval [1,5] and that f takes values shown in the table. The function f will have two zeros in the interval [1,5] if $k =$

A. –1
B. 0
C. 1
D. 2
E. 3

7. If $\frac{dy}{dt} = ky$ and $k \neq 0$, which of the following could be the equation of y?

A. $y = kx - 7$
B. $y = 95e^{kt}$
C. $y = 5 + \ln k$
D. $y = (x-k)^2$
E. $y = \sqrt[k]{x}$

8. If $F(x) = \int_1^x \sqrt{t^2 - t}\,dt$, then $F'(3) =$

A. 6
B. 5
C. $\sqrt{6}$
D. $\sqrt{5}$
E. $\sqrt{3}$

9. If $f(x) = \tan(e^{2x})$, then $f''(x) =$

A. $2e^{2x}\sec^2(e^{2x})$
B. $8e^{2x}\tan(e^{2x})$
C. $4e^{2x}\sec^2(e^{2x})$
D. $8e^{4x}\sec^2(e^{2x})\tan(e^{2x})$
E. $4e^{2x}\sec^2(e^{2x})[2e^{2x}\tan(e^{2x}) + 1]$

10. If $f(x) = \sec(3x)$, then $f'\left(\frac{3\pi}{4}\right) =$

A. $-3\sqrt{2}$
B. $-\frac{3\sqrt{2}}{2}$
C. $\frac{3}{2}$
D. $\frac{3\sqrt{2}}{2}$
E. $3\sqrt{2}$

11. If $f(x) = \begin{cases} x^3 e^x & \text{for } 0 \le x < 1 \\ \frac{e^x}{x^3} & \text{for } 1 < x \le 3 \end{cases}$, then $\lim_{x \to 1} f(x)$ is

A. 0
B. 1
C. e
D. e^3
E. nonexistent

12. If $f''(x) = x(x-2)(x+1)^2$, then the graph of f has points of inflection when $x =$

A. –2 and 1
B. 2 and –1
C. 2 and 0
D. –2 and 0
E. 0, 2 and –1

13. If $3x^2 - 4xy = 1$, then when $x = 1$, $\frac{dy}{dx} =$

A. $\frac{3}{2}$
B. 1
C. $\frac{1}{2}$
D. 0
E. $-\frac{1}{2}$

14. If a curve is defined by $f(x) = 1 - x\cos x$, an equation of the normal to the curve at $\left(\frac{\pi}{2}, 1\right)$ is

A. $y = \frac{\pi x}{2} + \frac{\pi}{2}$
B. $y = \frac{\pi x}{2} + \left(\frac{\pi}{2}\right)^2$
C. $y = \frac{2x}{\pi} - 2$
D. $y = -\frac{2x}{\pi} + 2$
E. $y = -\frac{2x}{\pi} - 2$

15. If a function is given by $f(x)=\frac{x+3}{x^2-1}$, what is the instantaneous rate of change of the function at $x = 3$?

A. $\frac{7}{16}$

B. $-\frac{7}{16}$

C. $\frac{11}{16}$

D. $-\frac{11}{16}$

E. $\frac{1}{6}$

16. If $f(x)$ is a twice differentiable function on the interval $a<x<b$, and $f'(x)=c$ for all $a<x<b$, then $\int_a^b f''(x)dx =$

A. $b-a$

B. $c(b-a)$

C. 0

D. $c(a-b)$

E. $a-b$

17. If $f(x)$ is continuous on the closed interval $[a,b]$ and differentiable on the open interval (a,b), which of the following must be true?

I. If $f(a)=f(b)$, then for some value c between a and b, $f'(c)=0$

II. If k is any number between $f(a)$ and $f(b)$, there is a value $c\in(a,b)$ such that $f(c)=k$.

III. There is a value $c\in(a,b)$ such that $f'(c)=\frac{f(b)-f(a)}{b-a}$.

A. I only

B. II only

C. III only

D. I and III

E. I, II, and III

18. Let f and g be differentiable functions such that $f(x)\neq 0$ and $g(2)=3$. If $h(x)=\frac{g(x)}{f(x)}$ and $h'(x)=\frac{-g(x)f'(x)}{[f(x)]^2}$, then $g(x)=$

A. $f(3)$

B. 0

C. $-\frac{[f(x)]^2}{f'(x)}$

D. 3

E. $f'(2)x+3$

19. Over the interval $0\le t\le 5$, the position of a particle is given by $s(t)=t^4-t^3-t+1$. What is the minimum velocity of the particle on the interval $0\le t\le 5$?

A. $t=\frac{1}{2}$

B. $t=\frac{1}{4}$

C. $t=1$

D. $t=-1$

E. $t=2$

20. The function f is given by $f(x)=2x^4-3x^2+1$. On which of the following intervals is f decreasing?

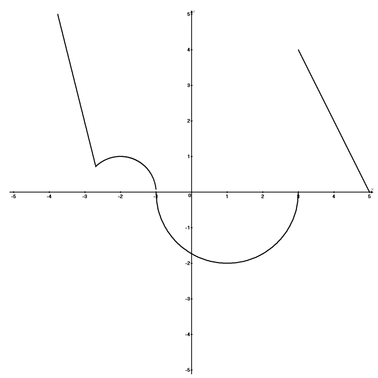
A. $\left(\frac{\sqrt{3}}{2},\infty\right)$

B. $\left(-\frac{\sqrt{3}}{2},\frac{\sqrt{3}}{2}\right)$

C. $\left(-\infty,-\frac{\sqrt{3}}{2}\right)$

D. $\left(0,\frac{\sqrt{3}}{2}\right)$

E. $\left(-\infty,-\frac{\sqrt{3}}{2}\right)$ and on $\left(0,\frac{\sqrt{3}}{2}\right)$



21. The function f shown in the graph above has horizontal tangents at (–2,1) and (1,–2) and vertical tangents at (–1,0) and (3,0). For how many values of x in the interval (–5,5) is the function not differentiable?

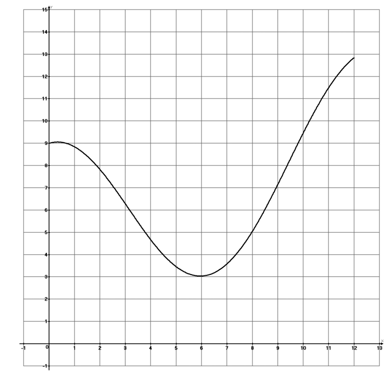
A. 0

B. 1

C. 2

D. 3

E. 4



22. The graph above shows the rate, in hundreds of passengers per hour, at which commuters passed through a subway station during a 12-hour period. Which of the following is the best estimate of the number of commuters who passed through the station in that 12-hour period?

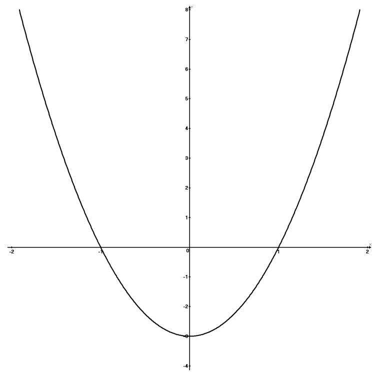
A. 840

B. 900

C. 1300

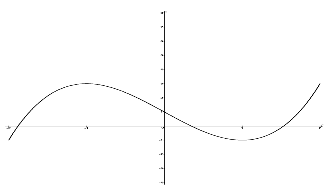
D. 8400

E. 9600

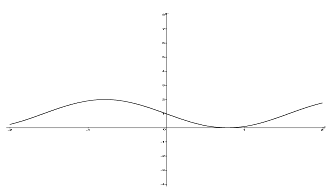


23. The graph of f', the derivative of f, is shown in the figure above. Which of the following could be the graph of f?

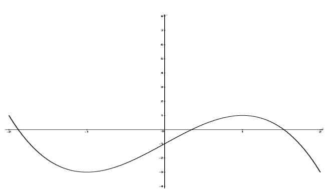
A.



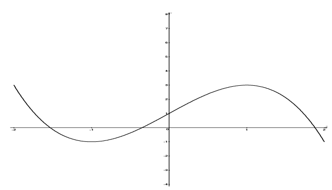
B.



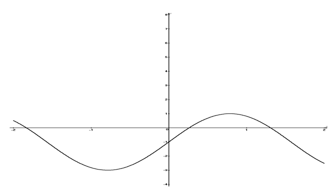
C.

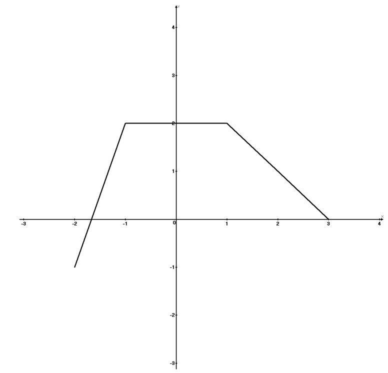


D.



E.





24. The graph of a piecewise linear function $f(x)$ for $-2 \le x \le 3$ is shown above. What is the value of $\int_{-2}^{3} f(x)dx$?

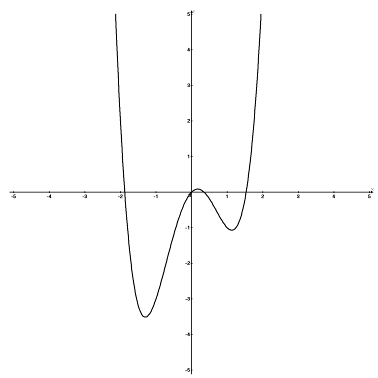
A. $\frac{13}{2}$

B. $\frac{41}{6}$

C. 8

D. 9

E. $\frac{37}{6}$



25. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

A. $f'(0) > f''(0) > f(0)$
B. $f(0) > f'(0) > f''(0)$
C. $f(0) > f''(0) > f'(0)$
D. $f''(0) > f(0) > f'(0)$
E. $f'(0) > f(0) > f''(0)$

26. What are all the values of k for which $\int_{-1}^{k} (x^2 - 2)dx = \frac{k^2}{2}$?

A. $k = -\frac{3}{2}$

B. $k = \sqrt{6}$

C. $k = \pm\sqrt{6}$

D. $k = \pm\sqrt{6}$ or $k = -\frac{3}{2}$

E. $k = \frac{3}{2}$

27. What is the area of the region enclosed by the graphs of $y = 2x^2 + 1$ and $y = x^3 - 1$ and the vertical lines $x = -1$ and $x = 2$?

A. 9

B. $\frac{99}{12}$

C. 7

D. $\frac{155}{12}$

E. 14

28. Which of the following is the x-coordinate of a point of inflection on the graph of $y = \frac{1}{4}x^4 - \frac{1}{2}x^2 + 3$?

A. $\frac{\sqrt{3}}{3}$

B. $-\frac{\sqrt{3}}{3}$

C. $\pm\frac{\sqrt{3}}{3}$

D. ± 1

E. -1

SECTION I, Part B

Time—50 Minutes

Number of Questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems. After examining the choices, select the best answer. No credit will be given for anything written in the test book.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the answer choices given. When this happens, select the answer that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. Which of the following is an equation of a tangent line to the graph of $2xy - x^2 = y$ at the point where $y = 1$?

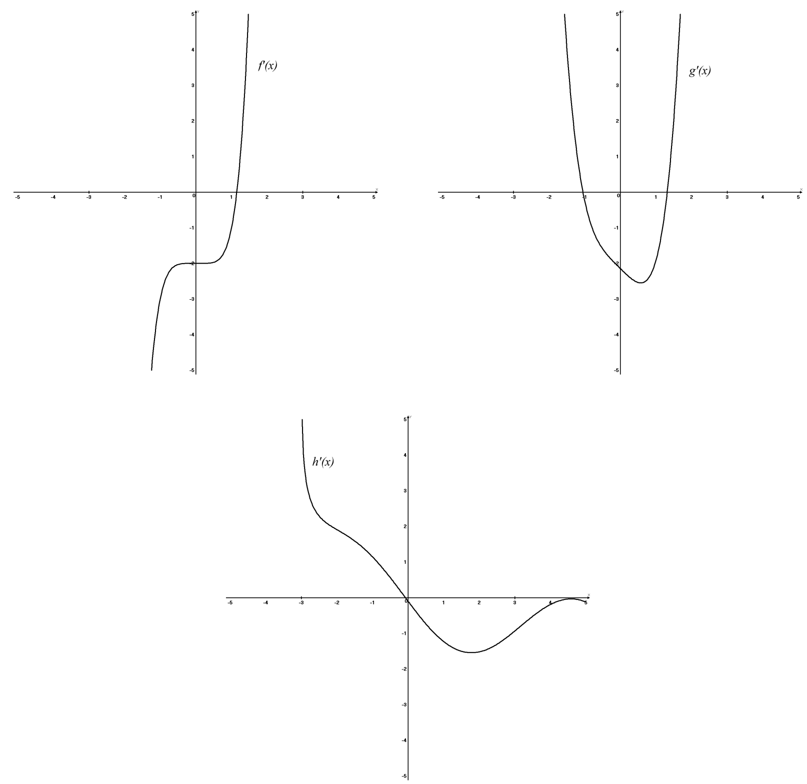
 A. $y = 2x - 1$
 B. $y = 2x - 3$
 C. $y = 1$
 D. $y = x$
 E. $y = x - 1$

30. The side of a square is increasing at a constant rate of 0.2 centimeters per second. In terms of the perimeter, P, of the square, what is the rate of change of the area of the square in square centimeters per second?

 A. $0.8P$
 B. $0.2P$
 C. $0.1P$
 D. $0.01P$
 E. $0.04P$

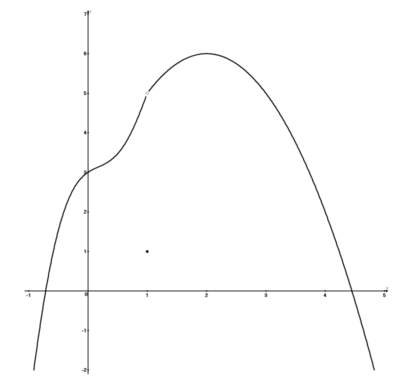
31. The population of bacteria in a culture grows at a rate that can be described by the equation $\frac{dy}{dt} = ky$, where y is the population and t is the time, measured in hours. If the population doubles every 3 hours, $k =$

 A. $\ln\left(\frac{2}{3}\right)$
 B. $\ln 3$
 C. $\ln 2$
 D. $\frac{\ln 2}{3}$
 E. 1



32. The graphs of the derivatives of functions f, g, and h are shown above. Which of the functions have a relative minimum on the interval $-3 < x < 3$?

A. g only
B. h only
C. f and g
D. g and h
E. f and h



33. The graph of a function f is shown above. Which of the following statements about f is true?

A. $x = 1$ is not in the domain of f.

B. f has a relative minimum at $x = 1$

C. $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x)$

D. $\lim_{x\to 1} f(x) = 1$

E. f is continuous at $x = 1$

x	1	2	3	4	5
$f(x)$	4	3	7	1	3

34. The function f is continuous on the closed interval [1,5] and values of the function are shown in the table above. If the values in the table are used to calculate a trapezoidal sum, the approximate value of $\int_1^5 f(x)dx$ is

A. 14
B. 14.5
C. 15
D. 29
E. 50

35. The first derivative of a function f is given by $f'(x) = \frac{\cos x(2x\sin x - \cos x)}{x^2}$. On the interval $0 < x < 8$, how many relative maxima does the function f have?

A. 0
B. 1
C. 2
D. 3
E. 4

36. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and the curve $x^2 + 2y = 4$. If the cross sections of the solid perpendicular to the x-axis are squares, the volume of the solid is

A. $\frac{72}{5}$
B. $\frac{128}{5}$
C. $\frac{192}{5}$
D. $\frac{56}{15}$
E. $\frac{64}{15}$

37. Let f and g be differentiable functions on the interval $(3,\infty)$ such that $g'(x) = f(x)\ln(x-3)$, and $f(x) > 0$ for all $x > 3$. Which of the following must be true?

A. $g(x)$ has a relative minimum at $x = 4$.
B. $g(x)$ has a relative maximum at $x = 3$.
C. $g(x)$ has a relative minimum at $x = 4$ and a relative maximum at $x = 3$.
D. $g(x)$ has no relative minimum or maximum.
E. There is not enough information to determine the relative extrema of $g(x)$.

38. Let f be the function given by $f(x) = (x-4)^2$ and let g be the function given by $g(x) = e^{3x}$. At what value of x do the graphs have tangent lines that are perpendicular?

A. $x \approx 1.143$
B. $x \approx -1.143$
C. $x \approx 0.512$
D. $x \approx -0.512$
E. $x \approx -0.750$

39. Let f be the function defined by $f(x) = x^2 - \frac{1}{x^3}$. Which of the following statements about f are true?

I. f is differentiable at $x = -1$.

II. f is continuous at $x = -1$.

III. f has an absolute maximum at $x = -1$.

A. I only
B. I and II
C. III only
D. II and III
E. I, II and III

40. Let f be a function that is differentiable on the open interval $(0,5)$. If $f(1)=2$, $f(3)=-1$, and $f(4)=5$, which of the following must be true?

I. For some value $1<c<4$, $f'(c)=1$.

II. The function f has at least three zeros on the interval $(0,5)$.

III. For some value $1<c<4$, $f(c)=4$.

A. I only
B. I and II
C. I and III
D. II and III
E. I, II and III

41. If the base of a triangle is increasing at a rate of 2 centimeters per minute, and its area remains constant, at what rate is the height changing?

A. $b-4h$
B. $-\frac{h}{4b}$
C. $-\frac{2h}{b}$
D. $\frac{4h}{b}$
E. $\frac{b}{4h}$

42. If $c \neq 0$, then $\lim_{x \to c} \frac{x^3-c^3}{x^2-c^2}$ is

A. 0
B. $\frac{3c}{2}$
C. $2c$
D. $2c^2$
E. nonexistent

43. If $F(x)$ is an antiderivative of $f(x)=\sqrt{4x+1}$ and $F(0)=1$, then $F(2)=$

A. $\frac{67}{3}$
B. $\frac{64}{3}$
C. $\frac{53}{3}$
D. $\frac{50}{3}$
E. $\frac{16}{3}$

44. If $\frac{\pi}{2}<k<\pi$ and the area under the curve $y=2\sin x$ from $x=\frac{\pi}{2}$ to $x=k$ is equal to 1.228, then $k=$

A. 2.617
B. 1.984
C. 1.797
D. 0.882
E. 0.117

45. $F(x)$ and $f(x)$ are continuous functions such that $F'(x)=f(x)$ for all x. $\int_2^5 f(3x)dx=$

A. $F(5)-F(2)$
B. $F(15)-F(6)$
C. $\frac{1}{3}[F(5)-F(2)]$
D. $3[F(5)-F(2)]$
E. $\frac{1}{3}[F(15)-F(6)]$

SECTION II

GENERAL INSTRUCTIONS

You may wish to look over the problems before you begin, since you may not complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- You may use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps.
- Unless otherwise specified, answers (numeric or algebraic) do not need to be simplified. If your answer is given as a decimal approximation, it should correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

SECTION II, PART A

Time—45 minutes

Number of problems—2

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS.

During the timed portion for Part A, you may work only on the problems in Part A.

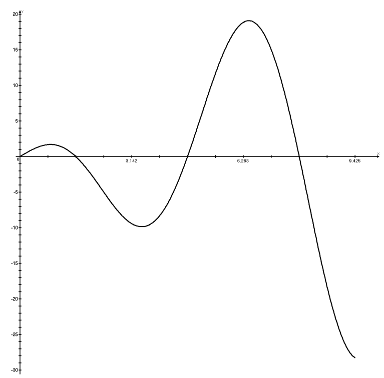
On Part A, you may use your calculator to solve an equation, find the derivative of a function at, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem. If you use other built-in features, you must show the steps necessary to produce your results.

1. Let R be the region bounded by $y = \dfrac{x^3 - 14x^2 + 53x - 40}{2x + 1}$ and the horizontal line $y = 3$, and let S be the region bounded by the graph of $y = \dfrac{x^3 - 14x^2 + 53x - 40}{2x + 1}$ and the horizontal lines $y = 1$ and $y = 3$.

a. Find the area of R.

b. Find the area of S.

c. Set up, but do not evaluate, an integral that could be used to find the volume of the solid generated when R is rotated about the horizontal line $y = 1$.



2. A particle moves along the x-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 3t\cos(t)$. The graph of v is shown above for $0 \leq t \leq 3\pi$. The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.

a. Find the acceleration of the particle at time $t = \pi$.

b. Find the total distance traveled by the particle from time $t = 0$ to $t = 3\pi$.

c. Find the position of the particle at time $t = \frac{3\pi}{2}$.

d. For $0 \leq t \leq 3\pi$, find the time t at which the particle is farthest to the left. Explain your answer.

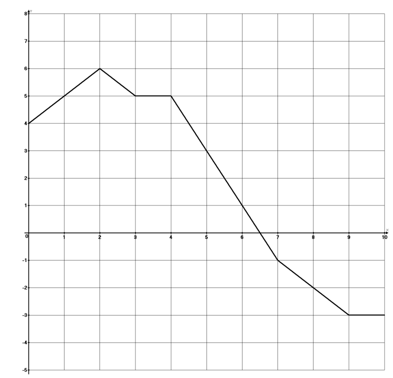
PART B

Time—60 minutes

Number of problems—4

NO CALCULATOR IS ALLOWED FOR THESE PROBLEMS.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.



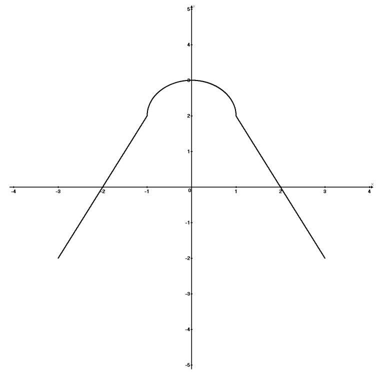
3. Jerry runs on a straight track starting at time $t = 0$ seconds and ending at time $t = 10$ seconds. During the time interval $0 \leq t \leq 10$, his velocity $v(t)$ in meters per second is modeled by the piecewise-linear function whose graph is shown above.

a. Find Jerry's acceleration at time $t = 4.5$ seconds. Indicate units of measure.

b. Using correct units, explain the meaning of $\int_0^{10} v(t)dt$ and how (if at all) it differs from $\int_0^{10} |v(t)|dt$.

c. At what time (if any) did Jerry change direction? Explain your reasoning.

d. Jeff runs on the same track, starting from the same starting line, with a velocity given by $f(t) = 5 - \frac{x^2}{25}$. At time $t = 10$ seconds, who is closer to the starting line: Jerry or Jeff?



4. Let f be a function defined on the closed interval $-3 \le x \le 3$ with $f(-1) = -2.8$ and $f(1) = 2.8$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown above.

a. For $-3 \le x \le 3$, find all values x at which f has a relative minimum. Justify your answer.

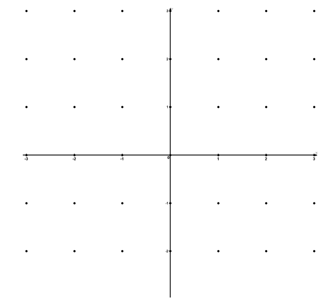
b. For $-3 \le x \le 3$, find all values x at which the graph of f has a point of inflection. Justify your answer.

c. Find all intervals on which the graph of f is increasing and concave down. Explain your reasoning.

d. Find the absolute maximum value of $f(x)$ over the closed interval $-3 \le x \le 3$. Explain your reasoning.

5. Consider the differential equation $\frac{dy}{dx} = xy - y$

a. On the axes provided, sketch a slope field for the given differential equation at the points indicated.



b. Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave down.

c. Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a critical point at $x = 1$? If so, describe the behavior of f at that point.

6. Let f be the function given by $f(x) = \frac{x^2 - x}{e^x}$ for all x.

a. Write an equation for the tangent to the graph of f at $x = 1$.

b. Find any critical points of f, and determine whether each is a relative maximum, relative minimum, or neither.

c. Find the x-coordinate of each point of inflection.

Solutions: AP Calculus AB Practice Test

Multiple Choice

Section I Part A

1. B. $\int_4^7 \frac{1}{(x-3)^2}dx = \int_4^7 (x-3)^{-2}dx = -(x-3)^{-1}\Big|_4^7 = \frac{-1}{7-3} - \frac{-1}{4-3} = -\frac{1}{4}+1 = \frac{3}{4}$

2. C. $\int_{\pi/2}^{x} \cos t\,dt = \sin t\,\Big|_{\pi/2}^{x} = \sin x - \sin\frac{\pi}{2} = \sin x - 1$

3. D. $\int_{1/e}^{e^2} \left(\frac{x^3+1}{x}\right)dx = \int_{1/e}^{e^2} \left(x^2+\frac{1}{x}\right)dx = \left[\frac{x^3}{3}+\ln x\right] =$

$\left(\frac{(e^2)^3}{3}+\ln e^2\right)-\left(\frac{(1/e)^3}{3}+\ln\frac{1}{e}\right) = \frac{e^6}{3}+2-\frac{1}{3e^3}+1 = \frac{e^9-1}{3e^3}+3 = \frac{e^9+9e^3-1}{3e^3}$

4. B. At the moment the particle changes direction, its velocity will be zero. Velocity is the derivative of position, so $v(t) = 5-\frac{2t}{3}$. Set velocity to zero, multiply through by 3, and solve $2t = 15$. Velocity is zero at $t = 7.5$ seconds, so the particle changes direction at 7.5 seconds.

5. D. The average value is $\frac{1}{3-1}\int_1^3 (x^3\sqrt{x^4-1})dx$. Use u-substitution with $u = x^4-1$ and $du = 4x^3dx$. Adjust the limits of integration. If $x = 1$, $u = 0$, and if $x = 3$, $u = 80$. Then the average value is

$= \frac{1}{3-1}\int_0^{80} \sqrt{u}\frac{du}{4} = \frac{1}{4(3-1)}\int_0^{80} \sqrt{u}\,du = \frac{1}{4(3-1)}\left[\frac{2}{3}u^{3/2}\right]_0^{80} = \frac{1}{8}\left[\frac{2}{3}(80)^{3/2}\right]$

$= \frac{1}{12}(4\sqrt{5})^3 = \frac{64\cdot 5\sqrt{5}}{12} = \frac{80\sqrt{5}}{3}$.

6. A. If f is continuous on [1,5] and has two zeros in the interval, then either f changes sign twice in the interval [1,5] or f has a relative max or relative min with a y-value of 0. If $k = 0$, the Intermediate Value Theorem guarantees that f will be equal to zero at least once in the interval [1,3] and at least once in the interval [3,5].

7. B. $\frac{dy}{dt} = ky$ and $k \neq 0$ should signal that this is a differential equation that leads to exponential growth, but if you don't immediately recognize that, separate the variables and take the antiderivative. If $\frac{dy}{y} = kdt$, then $\ln y = kt + c$ and $y = Ce^{kt}$.

8. C. If $F(x) = \int_1^x \sqrt{t^2-t}\,dt$, then by the Fundamental Theorem of Calculus, $F'(x) = \sqrt{x^2-x}$ and $F'(3) = \sqrt{9-3} = \sqrt{6}$.

9. E. If $f(x) = \tan(e^{2x})$, then $f'(x) = 2e^{2x}\sec^2(e^{2x})$ and

$f''(x) = 2e^{2x}\cdot 2\sec(e^{2x})\sec(e^{2x})\tan(e^{2x})\cdot 2e^{2x} + 4e^{2x}\sec^2(e^{2x})$

$= 8e^{4x}\sec^2(e^{2x})\tan(e^{2x}) + 4e^{2x}\sec^2(e^{2x}) = 4e^{2x}\sec^2(e^{2x})[2e^{2x}\tan(e^{2x})+1]$.

10. A. If $f(x) = \sec(3x)$, then $f'(x) = 3\sec(3x)\tan(3x)$, and

$f'\left(\frac{\pi}{4}\right) = 3\sec\frac{3\pi}{4}\tan\frac{3\pi}{4} = 3(-\sqrt{2})(1) = -3\sqrt{2}$.

11. C. If $f(x) = \begin{cases} x^3e^x & \text{for } 0 \le x < 1 \\ \frac{e^x}{x^3} & \text{for } 1 < x \le 3 \end{cases}$, then $\lim\limits_{x\to 1^-} f(x) = 1^3\cdot e^1 = e$ and $\lim\limits_{x\to 1^+} f(x) = \frac{e^1}{1^3} = e$, so $\lim\limits_{x\to 1} f(x) = e$.

12. C. If $f''(x) = x(x-2)(x+1)^2$, then $f''(x) = 0$ when $x = 0$, $x = 2$, and $x = -1$, but check for a change in concavity before deciding that all three values represent points of inflection.

x	$x < -2$	-1	$-1 < x < 0$	0	$0 < x < 2$	2	$x > 2$
$f''(x)$	+	0	+	0	−	0	+

The graph of f has points of inflection when $x = 0$ and when $x = 2$.

13. B. If $3x^2 4xy = 1$, differentiate implicitly. $6x - \left(4x\frac{dy}{dx} + 4y\right) = 0$ or $6x - 4y = 4x\frac{dy}{dx}$. Solve for $\frac{dy}{dx} = \frac{6x-4y}{4x} = \frac{3x-2y}{2x}$. Then when $x = 1$, $3 - 4y = 1$ so $2 = 4y$ and $y = \frac{1}{2}$. Evaluate $\frac{dy}{dx} = \frac{3\cdot 1 - 2\cdot\frac{1}{2}}{2\cdot 1} = \frac{2}{2} = 1$.

14. D. If a curve is defined by $f(x) = 1 - x\cos x$, $f'(x) = x\sin x - \cos x$ and $f'\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\sin\frac{\pi}{2} - \cos\frac{\pi}{2} = \frac{\pi}{2}$. The slope of the tangent is $\frac{\pi}{2}$, so the slope of the normal line is $m = -\frac{2}{\pi}$. Use point-slope form to show that an equation of the normal to the curve at $\left(\frac{\pi}{2}, 1\right)$ is $y - 1 = -\frac{2}{\pi}\left(x - \frac{\pi}{2}\right)$ or $y = -\frac{2x}{\pi} + 2$.

15. B. If a function is given by $f(x) = \frac{x+3}{x^2-1}$, the instantaneous rate of change of the function at $x = 3$ is $f'(x) = \frac{(x^2-1) - 2x(x+3)}{(x^2-1)^2} = \frac{x^2 - 1 - 2x^2 - 6x}{(x^2-1)^2} = -\frac{x^2+6x+1}{(x^2-1)^2}$, evaluate at $x = 3$.
$f'(3) = -\frac{9+18+1}{(9-1)^2} = -\frac{28}{64} = -\frac{7}{16}$.

16. C. If $f'(x) = c$ for all $a < x < b$, $f''(x) = 0$ for all $a < x < b$. Then $\int_a^b f''(x)dx = \int_a^b 0dx = 0x + C\Big|_a^b = 0$.

17. E. Rolle's Theorem guarantees that if $f(a) = f(b)$, then for some value c between a and b, $f'(c) = 0$. If k is any number between $f(a)$ and $f(b)$, the Intermediate Value Theorem assures that there is a value such that $c \in (a,b)$. According to the Mean Value Theorem, there is a value $c \in (a,b)$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

18. D. The derivative of $h(x) = \frac{g(x)}{f(x)}$ should be $h'(x) = \frac{f(x)g'(x) - g(x)f'(x)}{[f(x)]^2}$ but you are told $h'(x) = \frac{-g(x)f'(x)}{[f(x)]^2}$, so $f(x)g'(x) = 0$. Given that $f(x) \neq 0$, it must be that $g'(x) = 0$ and therefore g is a constant function. Since $g(2) = 3$, $g(x) = 3$.

19. A. If the position of a particle is given by $s(t) = t^4 - t^3 - t + 1$, then the velocity $v(t) = 4t^3 - 3t^2 - 1$ and the acceleration $a(t) = 12t^2 - 6t$. The minimum velocity of the particle on the interval $0 \le t \le 5$ will be attained when the acceleration is zero. $6t(2t-1) = 0$ when $t = 0$ or $t = \frac{1}{2}$. Check the sign of $a(t)$.

t	$t = 0$	$0 < t < \frac{1}{2}$	$t = \frac{1}{2}$	$t > \frac{1}{2}$
$a(t)$	0	–	0	+

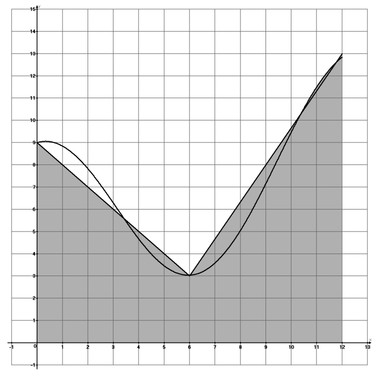
The velocity is a minimum when acceleration changes from negative to positive, at $t = \frac{1}{2}$.

20. E. Find the critical values of $f(x) = 2x^4 - 3x^2 + 1$ by finding $f'(x) = 8x^3 - 6x$ and solving $f'(x) = 8x^3 - 6x = 0$. $2x(4x^2 - 3) = 0$ so $x = 0$ and $x = \pm\frac{\sqrt{3}}{2}$. Test the sign on the derivative between critical values.

	$x < -\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2} < x < 0$	$0 < x < \frac{\sqrt{3}}{2}$	$x > \frac{\sqrt{3}}{2}$
$f'(x)$	–	+	–	+

The function f is decreasing on $\left(-\infty, -\frac{\sqrt{3}}{2}\right)$ and on $\left(0, \frac{\sqrt{3}}{2}\right)$ because the derivative is negative.

21. D. The function is not differentiable at the points where the tangent becomes vertical, $x = -1$ and $x = 3$, at the jump discontinuity, $x = 3$, and at the cusp, approximately $x = -2.7$.



22. D. The total number of commuters who passed through the station in the 12-hour period can be approximated by the area of the two trapezoids, as shown, using the points $(0,9)$, $(6,3)$, and $(12,13)$. $A = \frac{1}{2}\cdot 6(9+3) + \frac{1}{2}\cdot 6(3+13) = 36 + 48 = 84$. The best estimate of the number of commuters who passed through the station in that 12-hour period is 8400 commuters.

23. A. The graph of f' has zeros at $x = 1$ and $x = -1$, so the graph of f should have critical points at ± 1. At $x = -1$, the derivative changes from positive to negative, indicating a maximum, and at $x = 1$, the change from negative to positive indicates a minimum. This eliminates (C), (D), and (E). The end behavior of the graph of the derivative indicates that the graph of f increases to the right and decreases to the left at steeper and steeper rates. The graph of (B) shows a slowing in the rates of increase and decrease. Graph (A) is most likely to be the graph of f.

24. A. $\int_{-2}^{3} f(x)dx$ can be determined geometrically by calculating the area under the graph. The key is to determine the x-intercept between –2 and –1. The line segment connects $(-2,-1)$ to $(-1,2)$ and so has the equation $y - 2 = 3(x+1)$. Substituting 0 for y will give an x-intercept of $x = -\frac{5}{3}$. $\int_{-2}^{3} f(x)dx = \frac{1}{2}\cdot\frac{1}{3}(-1) + \frac{1}{2}\cdot 2\left(\frac{14}{3} + \frac{6}{3}\right) = -\frac{1}{6} + \frac{20}{3} = \frac{39}{6} = \frac{13}{2}$.

25. E. $f(0) = 0$. At $x = 0$, f is increasing so $f'(0) > 0$, and the graph of f is concave down, so $f''(0) < 0$. Therefore, $f'(0) > 0 > f''(0)$.

26. D. If $\int_{-3}^{k} (x^2 - 2)dx = \frac{k^2}{2}$, find the antiderivative. $\frac{x^3}{3} - 2x\big|_{-3}^{k} = \left(\frac{k^3}{3} - 2k\right) - (-9+6) = \frac{k^3}{3} - 2k + 3$. Then $\frac{k^3}{3} - 2k + 3 = \frac{k^2}{2}$ or $2k^3 - 12k + 18 = 3k^2$. Solve $2k^3 - 3k^2 - 12k + 18 = 0$ by factoring $k^2(2k-3) - 6(2k-3) = 0$, to find that $k = \pm\sqrt{6}$ or $k = \frac{3}{2}$.

27. B. The area of the region enclosed by the graphs of $y = 2x^2 + 1$ and $y = x^3 - 1$ and the vertical lines $x = -1$ and $x = 2$ is $\int_{-1}^{2} (2x^2+1) - (x^3-1)dx = \int_{-1}^{2} (-x^3 + 2x^2 + 2)dx = -\frac{1}{4}x^4 + \frac{2}{3}x^3 + 2x\big|_{-1}^{2} =$ $\left(-4 + \frac{16}{3} + 4\right) - \left(-\frac{1}{4} - \frac{2}{3} - 2\right) = \frac{99}{12}$.

28. C. Find the second derivative of $y = \frac{1}{4}x^4 - \frac{1}{2}x^2 + 3$. $y' = x^3 - x$ and $y'' = 3x^2 - 1$. Set the second derivative equal to zero and solve to find $x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$. Test for change in concavity.

x	$x < -\frac{\sqrt{3}}{3}$	$-\frac{\sqrt{3}}{3} < x < \frac{\sqrt{3}}{3}$	$x > \frac{\sqrt{3}}{3}$
$f''(x)$	+	–	+

The function has points of inflection at both $\pm\frac{\sqrt{3}}{3}$.

Section I Part B

29. C. At the point where $y = 1$, $2y - x^2 = y$ becomes $2x - x^2 = 1$ or $x^2 - 2x + 1 = 0$, so $x = 1$. Find the derivative. $2xy' + 2y - 2x = y'$ so $2xy' - y' = 2x - 2y$ and $y'(2x-1) = 2(x-y)$. Isolating $y' = \frac{2(x-y)}{2x-1}$. Evaluate at $x = 1$, $y = 1$ to find the slope of the tangent line is $m = \frac{2(1-1)}{2-1} = 0$. The equation of a tangent line to the graph of $2xy - x^2 = y$ at $(1,1)$ is $y - 1 = 0(x-1)$ or $y = 1$.

30. C. You know that $\frac{ds}{dt} = 0.2$. The area of the square is $A = s^2$ so $\frac{dA}{dt} = 2s\frac{ds}{dt}$. The perimeter, P, is $P = 4s$, so $s = \frac{P}{4}$ and $\frac{dA}{dt} = 2 \cdot \frac{P}{4}\frac{ds}{dt} = \frac{P}{2}(0.2) = 0.1P$.

31. D. If $\frac{dy}{dt} = ky$, $\frac{dy}{y} = kdt$ and $\ln y = kt + C$. This becomes the exponential growth equation $y = Ce^{kt}$. If the population doubles every 3 hours, $2 = 1e^{3k}$ so $\ln 2 = 3k$ and $k = \frac{\ln 2}{3}$.

32. C. The graph will have a relative minimum when the graph changes from decreasing to increasing, which is when the derivative changes from negative to positive. f' has one change of sign, from negative to positive, indicating that f has a relative minimum. g' has two changes of sign, indicating both a relative maximum and a relative minimum. h' has one sign change, but the change from positive to negative indicates a relative maximum, rather than a minimum.

33. C. The graph shows that the function is defined for $x = 1$, because $f(1) = 1$, but the graph does not change from decreasing to increasing at $x = 1$, so there is no relative minimum at $x = 1$. $\lim_{x\to 1^-} f(x) = 5$ and $\lim_{x\to 1^+} f(x) = 5$ so C is true, but D is not. Since $\lim_{x\to 1} f(x) = 5$ but $f(1) = 1$, the function is not continuous at $x = 1$.

x	1	2	3	4	5
$f(x)$	4	3	7	1	3

34. B. The function f is continuous on the closed interval $[1,5]$ and values of the function are shown in the table above. If the values in the table are used to calculate a trapezoidal sum, the approximate value of $\int_1^5 f(x)dx = \frac{1}{2}(4+3) + \frac{1}{2}(3+7) + \frac{1}{2}(7+1) + \frac{1}{2}(1+3)$ $= \frac{1}{2}(4 + 2\cdot 3 + 2\cdot 7 + 2\cdot 1 + 3) = \frac{1}{2}(4 + 6 + 14 + 2 + 3) = 14.5$.

35. D. The first derivative of a function f is given by $f'(x) = \frac{\cos x(2x\sin x - \cos x)}{x^2}$. To determine where the first derivative is zero, solve $\frac{\cos x(2x\sin x - \cos x)}{x^2} = 0$ or $\cos x(2x\sin x - \cos x) = 0$. Set each factor equal to zero. $\cos x = 0$ at $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ and $\cos x(2x\sin x - \cos x) = 0$ by using your calculator, when $x \approx 0.653$, $x \approx 3.292$, and $x \approx 6.362$. For relative maxima, you're looking for a change in the sign of the derivative from positive to negative, representing the function changing from increasing to decreasing. From the graphs, you can see that this occurs at $x = \frac{\pi}{2}$, $x = \frac{5\pi}{2}$, and $x \approx 3.292$. On the interval $0 < x < 8$, the function f has 3 relative maxima.

36. E. The base of a solid is a region in the first quadrant bounded by the x-axis, the y-axis, and $x^2 + 2y = 4$ or $y = \frac{4-x^2}{2}$. Each square cross section has a side equal to $s = \frac{4-x^2}{2}$ and therefore an area of $A = \left(\frac{4-x^2}{2}\right)^2$. Integrate to find the volume. $V = \int_0^2 \left(\frac{4-x^2}{2}\right)^2 dx = \frac{1}{4}\int_0^2 (16 - 8x^2 + x^4)dx$ $= \frac{1}{4}\left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_0^2 = \frac{1}{4}\left[32 - \frac{64}{3} + \frac{32}{5}\right] = \frac{64}{15}$.

37. A. The critical values of g occur when the derivative is zero. $g'(x) = f(x)\ln(x-3) = 0$ when $f(x) = 0$ or $\ln(x-3) = 0$. Since $f(x) > 0$, the relative extremum of g occurs when $\ln(x-3) = 0$, which is when $x - 3 = 1$, or $x = 4$. For values $3 < x < 4$, $g'(x) < 0$ and when $x > 4$, $g'(x) > 0$, which means that at $x = 4$, the graph of $g(x)$ changes from decreasing to increasing. $g(x)$ has a relative minimum at $x = 4$.

38. B. Find the derivatives $f'(x) = 2(x-4)$ and $g'(x) = 3e^{3x}$. If the tangent lines are perpendicular, the slopes are negative reciprocals, so solve $3e^{3x} = \frac{-1}{2(x-4)}$ by calculator to find $x \approx -1.143$.

39. B. If $f(x) = x^2 - \frac{1}{x^3}$, then $f'(x) = 2x + \frac{3}{x^4}$. The function is differentiable and continuous at $x = -1$. To determine where the relative extrema fall, solve $f'(x) = 2x + \frac{3}{x^4} = 0$. $2x = -\frac{3}{x^4}$ and $2x^5 = -3$ so $x = \sqrt[5]{-\frac{3}{2}}$. The extrema do not occur at $x = -1$.

40. C. If f is differentiable (and therefore continuous) on the open interval $(0,5)$, and $f(1)=2$, $f(3)=-1$, and $f(4)=5$, then the Mean Value Theorem guarantees that there is at least one value $1<c<4$ such that $f'(c)=\frac{f(4)-f(1)}{4-1}=\frac{5-2}{4-1}=1$, and the Intermediate Value Theorem guarantees that for any value $f(1)<k<f(4)$ there is some value $1<c<4$ such that $f(c)=k$. Therefore I is true and taking $k=1$, III is true. The Intermediate Value Theorem will also assure that there is a zero between $x=1$ and $x=3$, and a second zero between $x=3$ and $x=4$. There is no information to allow you to determine if there is a third zero in the interval.

41. C. The area of the triangle is $A=\frac{1}{2}bh$ so the area is changing at a rate equal to $\frac{dA}{dt}=\frac{1}{2}\left[b\frac{dh}{dt}+h\frac{db}{dt}\right]$. Since the area remains constant, $\frac{dA}{dt}=0$. Substitute $0=\frac{1}{2}\left[b\frac{dh}{dt}+2h\right]$ and solve for $\frac{dh}{dt}$. $\frac{1}{2}b\frac{dh}{dt}=-2h$ and $\frac{dh}{dt}=-\frac{2h}{b}$.

42. B. If $c\neq 0$, then $\lim\limits_{x\to c}\frac{x^3-c^3}{x^2-c^2}=\lim\limits_{x\to c}\frac{(x-c)(x^2+cx+c^2)}{(x-c)(x+c)}=\lim\limits_{x\to c}\frac{x^2+cx+c^2}{x+c}$ $=\frac{3c^2}{2c}=\frac{3c}{2}$.

43. E. By the Fundamental Theorem of Calculus, $\int_0^2 f(x)dx=F(2)-F(0)$. Find $\int_0^2\sqrt{4x+1}\,dx=\frac{1}{4}\cdot\frac{2}{3}(4x+1)^{3/2}\Big|_0^2=\frac{1}{6}[9^{3/2}-1^{3/2}]=\frac{1}{6}[27-1]=\frac{13}{3}$ and substitute $F(2)-F(0)=\frac{13}{3}$ or $F(2)=\frac{13}{3}+1=\frac{16}{3}$.

44. A. $\int_{\pi/2}^{k}(2-\sin x)dx=2x+\cos x\Big|_{\pi/2}^{k}=2k+\cos k-\pi-0$. You're given that the area under the curve from $x=\frac{\pi}{2}$ to $x=k$ is equal to 1.228, so $2k+\cos k-\pi=1.228$. Solve by calculator and $k\approx 2.617$.

45. E. If $u=3x$, $du=3dx$ or $dx=\frac{du}{3}$. When $x=2$, $u=6$ and when $x=5$, $u=15$. $\int_2^5 f(3x)dx=\frac{1}{3}\int_6^{15}f(u)du=\frac{1}{3}[F(15)-F(6)]$.

Section II Part A

1.

a. Use the calculator to find the intersection points of $y=\frac{x^3-14x^2+53x-40}{2x+1}$ and $y=3$, and then integrate. The area of R is $\int_{1.556}^{2.894}\left(\frac{x^3-14x^2+53x-40}{2x+1}-3\right)dx\approx 0.546$.

b. Use the calculator to find the intersection points of $y=\frac{x^3-14x^2+53x-40}{2x+1}$ and $y=1$. Find the area bounded by $y=\frac{x^3-14x^2+53x-40}{2x+1}$ and $y=1$.
$\int_{1.122}^{4.224}\left(\frac{x^3-14x^2+53x-40}{2x+1}-1\right)dx\approx 5.075$. Then subtract the area of region R. $5.075-0.546=4.529$

c. The outer radius is the distance from $y=1$ to $y=\frac{x^3-14x^2+53x-40}{2x+1}$, and the inner radius is the distance from $y=1$ to $y=3$. The volume of the solid generated when R is rotated about the horizontal line $y=1$ is given by $\pi\int_{1.556}^{2.894}\left[\left(1-\frac{x^3-14x^2+53x-40}{2x+1}\right)^2-(1-3)^2\right]dx$.

2.

a. The acceleration of the particle is given by $a(t)=v'(t)=3\cos(t)-3t\sin(t)$. At time $t=\pi$, $a(\pi)=3\cos(\pi)-3\pi\sin(\pi)=-3$.

b. The total distance traveled by the particle from time $t=0$ to $t=3\pi$ is $\int_0^{3\pi}|3t\cos t|dt=\int_0^{\pi/2}3t\cos t\,dt-\int_{\pi/2}^{3\pi/2}3t\cos t\,dt-\int_{5\pi/2}^{3\pi}3t\cos t\,dt$
$=1.712-(-18.850)+37.699-(-26.562)=84.823$.

c. The position of the particle at time $t=\frac{3\pi}{2}$ is
$x(0)+\int_0^{3\pi/2}3t\cos t\,dt=5-17.137=-12.137$.

d. The time t at which the particle is farthest to the left is the absolute minimum value of $x(t)$. First consider the critical points. From the graph of $v(t) = 3t\cos(t)$, you can see that velocity is zero at $t = \frac{\pi}{2}$, $t = \frac{3\pi}{2}$, and $t = \frac{5\pi}{2}$. At $t = \frac{\pi}{2}$, the change in the sign of the derivative is from positive to negative, indicating a relative maximum, and the same is true at $t = \frac{5\pi}{2}$. From part c, you know that $x\left(\frac{3\pi}{2}\right) = -12.137$ and the change of sign of the derivative from negative to positive indicates a relative minimum. Check the ends of the interval. $x(0) = 5$ and $x(3\pi) = x(0) + \int_0^{3\pi} 3t\cos t\,dt = 5 - 6 = -1$. The relative minimum at $t = \frac{3\pi}{2}$ is the absolute minimum of $x(t)$. The particle is farthest to the left at $t = \frac{3\pi}{2}$.

Section II, Part B

3.

a. Acceleration is the derivative of velocity, so his acceleration at time $t = 4.5$ seconds is the slope of the line segment connecting $(4,5)$ and $(7,-1)$. $m = \frac{-1-5}{7-4} = -\frac{6}{3} = -2$ meters per second.

b. $\int_0^{10} v(t)dt$ represents Jerry's displacement, or distance from the starting line, in meters, after 10 seconds, while $\int_0^{10} |v(t)|dt$ is the total distance Jerry travels, in meters, over the course of the ten seconds.

c. Jerry changes direction at 6.5 seconds, indicated by the velocity changing from positive to negative.

d. If Jeff's velocity is $f(t) = 5 - \frac{x^2}{25}$, his displacement is $\int_0^{10}\left(5 - \frac{x^2}{25}\right)dx = 5x - \frac{x^3}{75}\Big|_0^{10} = 50 - \frac{1000}{75} = 50 - \frac{40}{3} = \frac{100}{3} = 36\frac{2}{3}$ meters. Jerry's displacement can be found by the area under the curve. Using area calculations from geometry, $26.75 - 7.25 = 19.5$ meters. At time $t = 10$ seconds, Jerry is closer to the starting line.

4.

a. The function f will have a relative minimum when the derivative changes from negative to positive, which occurs at $x = -2$.

b. Point of inflection occur when concavity changes. We look for the points at which the second derivative changes sign, or when the derivative changes from increasing to decreasing or vice versa. The relative maximum of the derivative, at $x = 0$, represents such a change and therefore a point of inflection.

c. The graph of f is increasing when the derivative is positive, and concave down when the second derivative is negative or the first derivative is decreasing. The graph of the derivative is positive but decreasing from $x = 0$ to $x = 2$, so f is increasing and concave down on $0 < x < 2$.

d. From $x = -3$ to $x = -2$, f is decreasing. From $x = -2$ to $x = 2$, f is increasing, and from $x = 2$ to $x = 3$, it is decreasing. The absolute maximum is occurring either at $x = -3$ or $x = 2$. It is possible to determine the value of $f(-3)$ and $f(2)$ by integrating.

$$f'(x) = \begin{cases} 2x+4 & -3 \le x \le -1 \\ 2+\sqrt{1-x^2} & -1 < x < 1 \\ -2x+4 & 1 < x < 3 \end{cases} \text{ so } f(x) = \begin{cases} x^2+4x+c_1 & -3 \le x \le -1 \\ \int (2+\sqrt{1-x^2})dx & -1 < x < 1 \\ -x^2+4x+c_2 & 1 \le x \le 3 \end{cases}.$$

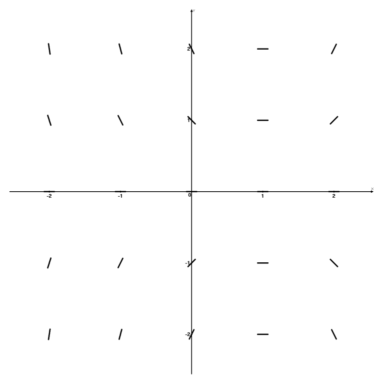
Since $f(-1) = -2.8$ and $f(1) = 2.8$, $f(x) = \begin{cases} x^2+4x+0.2 & -3 \le x \le -1 \\ \int (2+\sqrt{1-x^2})dx & -1 < x < 1 \\ -x^2+4x-0.2 & 1 \le x \le 3 \end{cases}.$

Then $f(-3) = (-3)^2 - 12 + 0.2 = -2.8$ and $f(2) = -4 + 8 - 0.2 = 3.8$. The absolute maximum occurs at $(2, 3.8)$.

5.

a. Calculate the value of $\frac{dy}{dx} = xy - y$ at each point and plot.

y / x	–2	–1	0	1	2
–2	6	3	0	–3	–6
–1	4	2	0	–2	–4
0	2	1	0	–1	–2
1	0	0	0	0	0
2	–2	–1	0	1	2



b. $\frac{d^2y}{dx^2} = x\frac{dy}{dx} + \frac{dy}{dx} = x(xy - y) + y - (xy - y) = x^2y - xy + y - xy + y$ $= x^2y - 2xy + 2y$. Solution curves to the differential equation are concave down when $\frac{d^2y}{dx^2} = x^2y - 2xy + 2y < 0$ or $y(x^2 - 2x + 2) < 0$. Since $x^2 - 2x + 2 > 0$ for all x, the second derivative is negative when $y < 0$.

c. At $x = 1$, $\frac{dy}{dx} = xy - y = y - y = 0$ so $x = 1$ is a critical point. Check the value of the derivative above and below $x = 1$. At $x = 0$, $\frac{dy}{dx} = -y$ and at $x = 2$, $\frac{dy}{dx} = 2y - y = y$. This is adequate to tell you that there is a change of sign, so the point is a relative of extremum of some kind, but further investigation is necessary to determine if it is a maximum or minimum. If $\frac{dy}{dx} = xy - y$, then $\frac{dy}{y} = (x-1)dx$ and $\ln y = \frac{1}{2}x^2 - x + c$. Since $f(0) = 1$, $\ln 1 = c$ and $c = 0$. Therefore $f(x) = e^{\frac{x^2}{2} - x}$ and $f'(x) = e^{\frac{x^2}{2} - x}(x-1)$. $f'(0) = -1$ and $f'(2) = 1$ so $x = 1$ is a relative minimum.

6. Let f be the function given by $f(x) = \frac{x^2 - x}{e^x}$ for all x.

a. Find the derivative $f'(x) = \frac{e^x(2x-1) - (x^2 - x)e^x}{(e^x)^2} = \frac{e^x(-x^2 + 3x - 1)}{e^{2x}}$ $= \frac{-x^2 + 3x - 1}{e^x}$. Evaluate $f(1) = \frac{1^2 - 1}{e^1} = 0$ and $f(1) = \frac{-1^2 + 3 - 1}{e^1} = \frac{1}{e}$.

The equation for the tangent to the graph of f at $x = 1$ is $y - 0 = \frac{1}{e}(x - 1)$ or $y = \frac{x-1}{e}$.

b. To find any critical points of f, set $f'(x) = \frac{-x^2 + 3x - 1}{e^x} = 0$ and solve. Since $e^x > 0$ for all x, solve $-x^2 + 3x - 1 = 0$ to find that the critical numbers are $x = \frac{-3 \pm \sqrt{9 - 4(-1)(-1)}}{2(-1)} = \frac{-3 \pm \sqrt{5}}{-2} = \frac{3 \pm \sqrt{5}}{2}$. Do a first derivative test for determine the behavior of f at each point.

x	0	$\frac{3-\sqrt{5}}{2}$	1	$\frac{3+\sqrt{5}}{2}$	3
$f'(x)$	–	0	+	0	–

The function f has a relative minimum at $x = \frac{3-\sqrt{5}}{2}$ and a relative maximum at $x = \frac{3+\sqrt{5}}{2}$.

c. Find the x-coordinate of each point of inflection, find the second derivative. $f''(x) = \frac{e^x(-2x+3) - e^x(-x^2 + 3x - 1)}{(e^x)^2} = \frac{e^x(x^2 - 5x + 4)}{e^{2x}}$ $= \frac{(x-1)(x-4)}{e^x}$. The second derivative will be zero at $x = 1$ and $x = 4$. Examine the sign of the second derivative to verify that there is a change in the concavity of the graph of f.

x	0	1	2	4	5
$f''(x)$	+	0	–	0	+

The graph of f has inflection points at $x = 1$ and $x = 4$.