Chapter 7 Summary
Scatterplots, Association, and Correlation

What have we learned?

We examine scatterplots for *direction*, *form*, *strength*, and *unusual features*.

Although not every relationship is linear, when the scatterplot is straight enough, the *correlation coefficient* is a useful numerical summary.

- The sign of the correlation tells us the direction of the association.
- The magnitude of the correlation tells us the *strength* of a linear association.
- Correlation has no units, so shifting or scaling the data, standardizing, or swapping the variables has no effect on the numerical value.

Doing Statistics right means that we have to *Think* about whether our choice of methods is appropriate.

- Before finding or talking about a correlation, check the Straight Enough Condition.
- Watch out for outliers!

Don't assume that a high correlation or strong association is evidence of a cause-and-effect relationship—beware of lurking variables!

Looking at Scatterplots

Scatterplots may be the most common and most effective display for data.

In a scatterplot, you can see patterns, trends, relationships, and even the occasional extraordinary value sitting apart from the others.

Scatterplots are the best way to start observing the relationship and the ideal way to picture associations between two *quantitative* variables.

When looking at scatterplots, we will look for direction, form, strength, and unusual features.

Direction:

A pattern that runs from the upper left to the lower right is said to have a negative direction.

A trend running the other way has a positive direction.

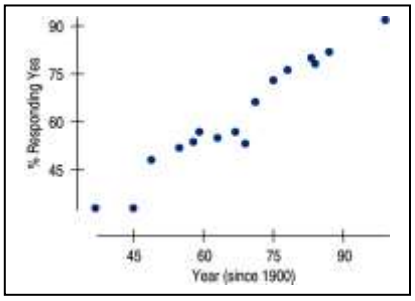


Figure 7.1 from the text shows a positive association between the year since 1900 and the % of people who say they would vote for a woman president.

As the years have passed, the percentage who would vote for a woman has increased.

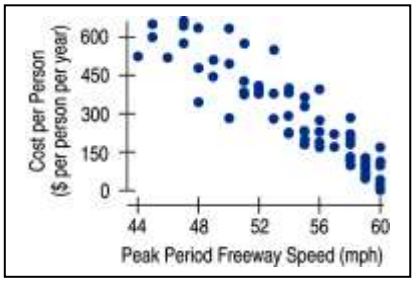


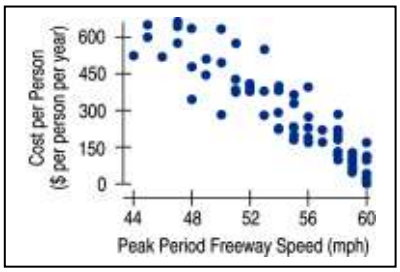
Figure 7.2 from the text shows a negative association between peak period freeway speed and cost per person of traffic delays.

As the peak period freeway speed increases, the cost per person of traffic delays decreases.

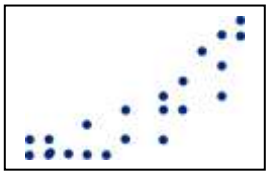
Looking at Scatterplots

Form:

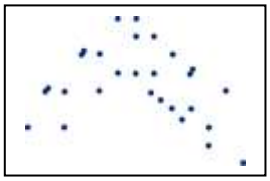
If there is a straight line (linear) relationship, it will appear as a cloud or swarm of points stretched out in a generally consistent, straight form.



If the relationship isn't straight, but curves gently, while still increasing or decreasing steadily, we can often find ways to make it more nearly straight.



If the relationship curves sharply, the methods of this book cannot really help us.



Strength:

At one extreme, the points appear to follow a single stream (whether straight, curved, or bending all over the place).

At the other extreme, the points appear as a vague cloud with no discernable trend or pattern:

Note: we will quantify the amount of scatter soon.

Unusual features:

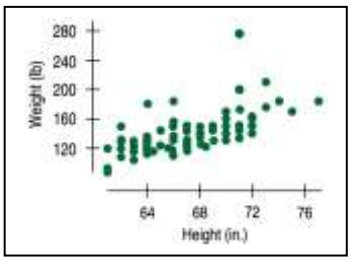
- Look for the unexpected.
- Often the most interesting thing to see in a scatterplot is the thing you never thought to look for.
- One example of such a surprise is an outlier standing away from the overall pattern of the scatterplot.
- Clusters or subgroups should also raise questions.

Roles for Variables

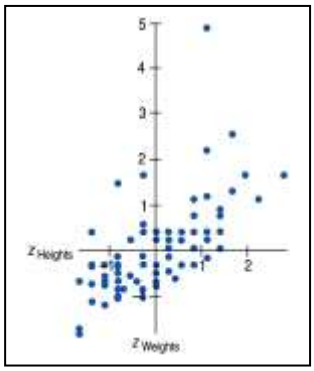
- It is important to determine which of the two quantitative variables goes on the *x*-axis and which on the *y*-axis.
- This determination is made based on the roles played by the variables.
- When the roles are clear, the explanatory or predictor variable goes on the *x*-axis, and the response variable goes on the *y*-axis.
- The roles that we choose for variables are more about how we *think* about them rather than about the variables themselves.
- Just placing a variable on the *x*-axis doesn't necessarily mean that it explains or predicts *anything*. And the variable on the *y*-axis may not respond to it in any way.

Correlation

- Data collected from students in Statistics classes included their heights (in inches) and weights (in pounds)
- Here we see a positive association and a fairly straight form, although there seems to be a high outlier.
- How strong is the association between weight and height of Statistics students?
- If we had to put a number on the strength, we would not want it to depend on the units we used.
- A scatterplot of heights (in centimeters) and weights (in kilograms) doesn't change the shape of the pattern

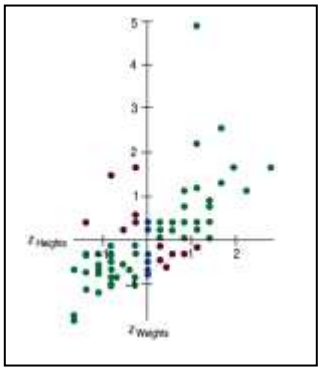


- Since the units don't matter, why not remove them altogether?
- We could standardize both variables and write the coordinates of a point as (z_x, z_y).
- Here is a scatterplot of the standardized weights and heights:
- Note that the underlying linear pattern seems steeper in the standardized plot than in the original scatterplot.
- That's because we made the scales of the axes the same.
- Equal scaling gives a neutral way of drawing the scatterplot and a fairer impression of the strength of the association.



- Some points (those in green) strengthen the impression of a positive association between height and weight.
- Other points (those in red) tend to weaken the positive association.
- Points with *z*-scores of zero (those in blue) don't vote either way.
- The correlation coefficient (r) gives us a numerical measurement of the strength of the linear relationship between the explanatory and response variables.

$$r = \frac{\sum z_x z_y}{n-1}$$



- For the students' heights and weights, the correlation is 0.644.
- What does this mean in terms of strength? We'll address this shortly.

Correlation Conditions

Correlation measures the strength of the *linear* association between two *quantitative* variables.

Before you use correlation, you must check several conditions:

- Quantitative Variables Condition
- Straight Enough Condition
- Outlier Condition

Quantitative Variables Condition:

- Correlation applies only to quantitative variables.
- Don't apply correlation to categorical data masquerading as quantitative.
- Check that you know the variables' units and what they measure.

Correlation Conditions (cont.)

Straight Enough Condition:

- You can *calculate* a correlation coefficient for any pair of variables.
- But correlation measures the strength only of the *linear* association, and will be misleading if the relationship is not linear.

Outlier Condition:

- Outliers can distort the correlation dramatically.
- An outlier can make an otherwise small correlation look big or hide a large correlation.
- It can even give an otherwise positive association a negative correlation coefficient (and vice versa).
- When you see an outlier, it's often a good idea to report the correlations with and without the point.

Correlation Properties

The sign of a correlation coefficient gives the direction of the association.

Correlation is always between -1 and +1.

Correlation *can* be exactly equal to -1 or +1, but these values are unusual in real data because they mean that all the data points fall *exactly* on a single straight line.

A correlation near zero corresponds to a weak linear association.

Correlation treats *x* and *y* symmetrically:

The correlation of *x* with *y* is the same as the correlation of *y* with *x*.

Correlation has no units.

Correlation is not affected by changes in the center or scale of either variable.

Correlation depends only on the *z*-scores, and they are unaffected by changes in center or scale.

Correlation measures the strength of the *linear* association between the two variables.

Variables can have a strong association but still have a small correlation if the association isn't linear.

Correlation is sensitive to outliers. A single outlying value can make a small correlation large or make a large one small.

Correlation Tables

It is common in some fields to compute the correlations between each pair of variables in a collection of variables and arrange these correlations in a table.

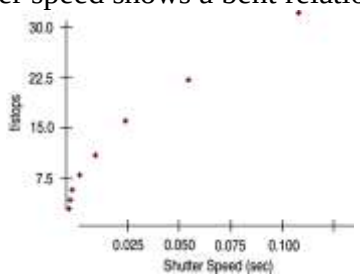
	Assets	Sales	Market Value	Profits	Cash Flow	Employees
Assets	1.000					
Sales	0.746	1.000				
Market Value	0.682	0.879	1.000			
Profits	0.602	0.814	0.968	1.000		
Cash Flow	0.641	0.855	0.970	0.989	1.000	
Employees	0.594	0.924	0.818	0.762	0.787	1.000

*Straightening Scatterplots

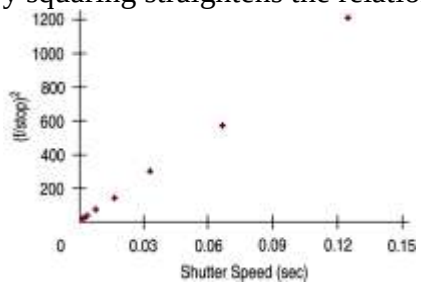
Straight line relationships are the ones that we can measure with correlation.

When a scatterplot shows a bent from that consistently increases or decreases, we can often straighten the form of the plot by re-expressing one or both variables.

A scatterplot of f/stop vs. shutter speed shows a bent relationship:



Re-expressing f/stop speed by squaring straightens the relationship:



What Can Go Wrong?

- Don't say "correlation" when you mean "association."
  - More often than not, people say correlation when they mean association.
  - The word "correlation" should be reserved for measuring the strength and direction of the linear relationship between two quantitative variables.
- Don't correlate categorical variables.
  - Be sure to check the Quantitative Variables Condition.
- Be sure the association is linear.
  - There may be a strong association between two variables that have a nonlinear association.
- Beware of outliers.
  - Even a single outlier can dominate the correlation value.
  - Make sure to check the Outlier Condition.
- Don't confuse correlation with causation.
  - Not every relationship is one of cause and effect.
- Watch out for lurking variables.
  - A hidden variable that stands behind a relationship and determines it by simultaneously affecting the other two variables is called a lurking variable.