Chapter 17 Summary
Probability Models

What have we learned?

- Bernoulli trials show up in lots of places.
- Depending on the random variable of interest, we might be dealing with a
  - Geometric model
  - Binomial model
  - Normal model
- Geometric model
  - When we're interested in the number of Bernoulli trials until the next success.
- Binomial model
  - When we're interested in the number of successes in a certain number of Bernoulli trials.
- Normal model
  - To approximate a Binomial model when we expect at least 10 successes and 10 failures.

Bernoulli Trials

- The basis for the probability models we will examine in this chapter is the Bernoulli trial.
- We have Bernoulli trials if:
  - there are two possible outcomes (success and failure).
  - the probability of success, p, is constant.
  - the trials are independent.

The Geometric Model

- A single Bernoulli trial is usually not all that interesting.
- A Geometric probability model tells us the probability for a random variable that counts the number of Bernoulli trials until the first success.
- Geometric models are completely specified by one parameter, p, the probability of success, and are denoted Geom(p).
- Geometric probability model for Bernoulli trials: Geom(p)
  - p = probability of success
  - $q = 1 - p$ = probability of failure
  - X = # of trials until the first success occurs

$$P(X = x) = q^{x-1}p \qquad \mu = \frac{1}{p} \qquad \sigma = \sqrt{\frac{q}{p^2}}$$

Independence

- One of the important requirements for Bernoulli trials is that the trials be independent.
- When we don't have an infinite population, the trials are not independent. But, there is a rule that allows us to pretend we have independent trials:
  - The 10% condition: Bernoulli trials must be independent. If that assumption is violated, it is still okay to proceed as long as the sample is smaller than 10% of the population.

The Binomial Model

- A Binomial model tells us the probability for a random variable that counts the number of successes in a fixed number of Bernoulli trials.
- Two parameters define the Binomial model: *n*, the number of trials; and, *p*, the probability of success. We denote this Binom(*n*, *p*).
- In *n* trials, there are ${}_nC_k = \frac{n!}{k!(n-k)!}$ ways to have *k* successes. Read ${}_nC_k$ as "*n* choose *k*."
- Note: $n! = n \times (n-1) \times \ldots \times 2 \times 1$, and *n!* is read as "*n* factorial."
- Binomial probability model for Bernoulli trials: Binom(n,*p*)
  - *n* = number of trials
  - *p* = probability of success
  - $q = 1 - p$ = probability of failure
  - *X* = # of successes in *n* trials

$$P(X = x) = {}_nC_x\, p^x q^{n-x} \qquad \mu = np \qquad \sigma = \sqrt{npq}$$

The Normal Model to the Rescue!

- When dealing with a large number of trials in a Binomial situation, making direct calculations of the probabilities becomes tedious (or outright impossible).
- Fortunately, the Normal model comes to the rescue…
- As long as the Success/Failure Condition holds, we can use the Normal model to approximate Binomial probabilities.
  - Success/failure condition: A Binomial model is approximately Normal if we expect at least 10 successes and 10 failures: $np \geq 10$ and $nq \geq 10$

Continuous Random Variables

- When we use the Normal model to approximate the Binomial model, we are using a continuous random variable to approximate a discrete random variable.
- So, when we use the Normal model, we no longer calculate the probability that the random variable equals a *particular* value, but only that it lies *between* two values.

What Can Go Wrong?

- Be sure you have Bernoulli trials.
  - You need two outcomes per trial, a constant probability of success, and independence.
  - Remember that the 10% Condition provides a reasonable substitute for independence.
- Don't confuse Geometric and Binomial models.
- Don't use the Normal approximation with small *n*.
  - You need at least 10 successes and 10 failures to use the Normal approximation.