# Chapter 16 Summary
## *Random Variables*

*What have we learned?*

- We know how to work with random variables.
  - We can use a probability model for a discrete random variable to find its expected value and standard deviation.
- The mean of the sum or difference of two random variables, discrete or continuous, is just the sum or difference of their means.
- And, *for independent random variables*, the variance of their sum or difference is always the *sum* of their variances.
- Normal models are once again special.
  - Sums or differences of Normally distributed random variables also follow Normal models.

Expected Value: Center

- A random variable assumes a value based on the outcome of a random event.
  - We use a capital letter, like *X*, to denote a random variable.
  - A particular value of a random variable will be denoted with a lower case letter, in this case *x*.
- There are two types of random variables:
  - Discrete random variables can take one of a finite number of distinct outcomes.
    - Example: Number of credit hours
  - Continuous random variables can take any numeric value within a range of values.
    - Example: Cost of books this term
- A probability model for a random variable consists of:
  - The collection of all possible values of a random variable, and
  - the probabilities that the values occur.
- Of particular interest is the value we expect a random variable to take on, notated $\mu$ (for population mean) or *E(X)* for expected value.
- The expected value of a (discrete) random variable can be found by summing the products of each possible value by the probability that it occurs: $\mu = E(X) = \sum x \cdot P(X = x)$
- Note: Be sure that every possible outcome is included in the sum and verify that you have a valid probability model to start with.

First Center, Now Spread…

- For data, we calculated the standard deviation by first computing the deviation from the mean and squaring it. We do that with random variables as well.
- The variance for a random variable is: $\sigma^2 = Var(X) = \sum (x - \mu)^2 \cdot P(X = x)$
- The standard deviation for a random variable is: $\sigma = SD(X) = \sqrt{Var(X)}$

More About Means and Variances

- Adding or subtracting a constant from data shifts the mean but doesn't change the variance or standard deviation:

$$E(X \pm c) = E(X) \pm c \qquad Var(X \pm c) = Var(X)$$

  - Example: Consider everyone in a company receiving a $5000 increase in salary.

More About Means and Variances (cont.)

- In general, multiplying each value of a random variable by a constant multiplies the mean by that constant and the variance by the *square* of the constant:

$$E(aX) = aE(X) Var(aX) = a^2 Var(X)$$

  - Example: Consider everyone in a company receiving a 10% increase in salary.
- In general,
  - The mean of the sum of two random variables is the sum of the means.
  - The mean of the difference of two random variables is the difference of the means. $E(X \pm Y) = E(X) \pm E(Y)$
  - If the random variables are *independent*, the variance of their sum *or* difference is always the sum of the variances. $Var(X \pm Y) = Var(X) + Var(Y)$

Continuous Random Variables

- Random variables that can take on any value in a range of values are called continuous random variables.
- Continuous random variables have means (expected values) and variances.
- We won't worry about how to calculate these means and variances in this course, but we can still work with models for continuous random variables when we're given the parameters.
- Good news: nearly everything we've said about how discrete random variables behave is true of continuous random variables, as well.
- When two independent continuous random variables have Normal models, so does their sum or difference.
- This fact will let us apply our knowledge of Normal probabilities to questions about the sum or difference of independent random variables.

*What Can Go Wrong?*

- Probability models are still just models.
  - Models can be useful, but they are not reality.
  - Question probabilities as you would data, and think about the assumptions behind your models.
- If the model is wrong, so is everything else.
- Don't assume everything's Normal.
- Watch out for variables that aren't independent:
  - You can add expected values for *any* two random variables, but
  - you can only add variances of *independent* random variables.
- Don't forget: Variances of independent random variables add. Standard deviations don't.
- Don't forget: Variances of independent random variables add, even when you're looking at the difference between them.
- Don't forget: Don't write independent instances of a random variable with notation that looks like they are the same variables.