Chapter 15 Summary
Probability Rules!

What have we learned?

- The probability rules from Chapter 14 only work in special cases—when events are disjoint or independent.
- We now know the General Addition Rule and General Multiplication Rule.
- We also know about conditional probabilities and that reversing the conditioning can give surprising results.
- Venn diagrams, tables, and tree diagrams help organize our thinking about probabilities.
- We now know more about independence—a sound understanding of independence will be important throughout the rest of this course.

Recall That…

- For any random phenomenon, each trial generates an outcome.
- An event is *any* set or collection of outcomes.
- The collection of *all possible* outcomes is called the sample space, denoted **S**.

Events

- When outcomes are *equally likely*, probabilities for events are easy to find just by counting.
- When the *k* possible outcomes are equally likely, each has a probability of 1/*k*.
- For any event **A** that is made up of equally likely outcomes, $P(\mathbf{A}) = \dfrac{\text{count of outcomes in } \mathbf{A}}{\text{count of all possible outcomes}}$

The First Three Rules for Working with Probability Rules

- Make a picture, Make a picture, Make a picture.

Picturing Probabilities

- The most common kind of picture to make is called a Venn diagram.
- We will see Venn diagrams in practice shortly…

The General Addition Rule

- When two events **A** and **B** are disjoint, we can use the addition rule for disjoint events from Chapter 14: $P(\mathbf{A} \text{ or } \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B})$
- However, when our events are not disjoint, this earlier addition rule will double count the probability of *both* **A** and **B** occurring. Thus, we need the General Addition Rule.
- General Addition Rule: For any two events **A** and **B**,
$$P(\mathbf{A} \text{ or } \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B}) - P(\mathbf{A} \text{ and } \mathbf{B})$$

It Depends…

- Back in Chapter 3, we looked at contingency tables and talked about conditional distributions.
- When we want the probability of an event from a *conditional* distribution, we write $P(\mathbf{B}|\mathbf{A})$ and pronounce it "the probability of **B** *given* **A**."
- A probability that takes into account a given condition is called a conditional probability.

It Depends… (cont.)

- To find the probability of the event **B** *given* the event **A**, we restrict our attention to the outcomes in **A**. We then find the fraction of *those* outcomes **B** that also occurred.

$$P(\mathbf{B}|\mathbf{A})=\frac{P(\text{A and B})}{P(\mathbf{A})}$$

- Note: $P(\mathbf{A})$ cannot equal 0, since we know that **A** has occurred.

The General Multiplication Rule

- When two events **A** and **B** are independent, we can use the multiplication rule for independent events from Chapter 14: $P(\mathbf{A}\ \textit{and}\ \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$
- However, when our events are not independent, this earlier multiplication rule does not work. Thus, we need the General Multiplication Rule.
- We encountered the general multiplication rule in the form of conditional probability.
- Rearranging the equation in the definition for conditional probability, we get the General Multiplication Rule: For any two events **A** and **B**,

$$P(\mathbf{A}\ \textit{and}\ \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B}|\mathbf{A})$$

or

$$P(\mathbf{A}\ \textit{and}\ \mathbf{B}) = P(\mathbf{B}) \times P(\mathbf{A}|\mathbf{B})$$

Independence

- Independence of two events means that the outcome of one event does not influence the probability of the other.
- With our new notation for conditional probabilities, we can now formalize this definition:
    - Events **A** and **B** are independent whenever $P(\mathbf{B}|\mathbf{A}) = P(\mathbf{B})$. (Equivalently, events **A** and **B** are independent whenever $P(\mathbf{A}|\mathbf{B}) = P(\mathbf{A})$.)

Independent ≠ Disjoint

- Disjoint events *cannot* be independent! Well, why not?
    - Since we know that disjoint events have no outcomes in common, knowing that one occurred means the other didn't.
    - Thus, the probability of the second occurring changed based on our knowledge that the first occurred.
    - It follows, then, that the two events are *not* independent.
- A common error is to treat disjoint events as if they were independent, and apply the Multiplication Rule for independent events—don't make that mistake.

Depending on Independence

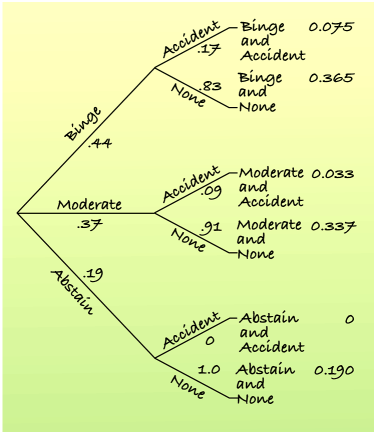
- It's much easier to think about independent events than to deal with conditional probabilities.
    - It seems that most people's natural intuition for probabilities breaks down when it comes to conditional probabilities.
- Don't fall into this trap: whenever you see probabilities multiplied together, stop and ask whether you think they are really independent.

Drawing Without Replacement

- Sampling without replacement means that once one individual is drawn it doesn't go back into the pool.
    - We often sample without replacement, which doesn't matter too much when we are dealing with a large population.
    - However, when drawing from a small population, we need to take note and adjust probabilities accordingly.
- Drawing without replacement is just another instance of working with conditional probabilities.

Tree Diagrams

- A tree diagram helps us think through conditional probabilities by showing sequences of events as paths that look like branches of a tree.
- Making a tree diagram for situations with conditional probabilities is consistent with our "make a picture" mantra.
- Figure 15.4 is a nice example of a tree diagram and shows how we multiply the probabilities of the branches together:



Reversing the Conditioning

- Reversing the conditioning of two events is rarely intuitive.
- Suppose we want to know $P(\mathbf{A}|\mathbf{B})$, but we know only $P(\mathbf{A})$, $P(\mathbf{B})$, and $P(\mathbf{B}|\mathbf{A})$.
- We also know $P(\mathbf{A} \textit{ and } \mathbf{B})$, since $P(\mathbf{A} \textit{ and } \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B}|\mathbf{A})$
- From this information, we can find $P(\mathbf{A}|\mathbf{B})$: $P(\text{A}|\text{B}) = \dfrac{P(\text{A and B})}{P(\text{B})}$
- When we reverse the probability from the conditional probability that you're originally given, you are actually using Bayes's Rule.

What Can Go Wrong?

- Don't use a simple probability rule where a general rule is appropriate:
    - Don't assume that two events are independent or disjoint without checking that they are.
- Don't find probabilities for samples drawn without replacement as if they had been drawn with replacement.
- Don't reverse conditioning naively.
- Don't confuse "disjoint" with "independent."