Chapter 14 Summary
From Randomness to Probability

What have we learned?

- Probability is based on long-run relative frequencies.
- The Law of Large Numbers speaks only of long-run behavior.
    - Watch out for misinterpreting the LLN.
- There are some basic rules for combining probabilities of outcomes to find probabilities of more complex events. We have the:
    - Something Has to Happen Rule
    - Complement Rule
    - Addition Rule for disjoint events
    - Multiplication Rule for independent events

Dealing with Random Phenomena

- A random phenomenon is a situation in which we know what outcomes could happen, but we don't know which particular outcome did or will happen.

Probability

- The probability of an event is its long-run relative frequency.
    - While we may not be able to predict a *particular* individual outcome, we can talk about what will happen *in the long run*.
- For any random phenomenon, each attempt, or trial, generates an outcome.
    - Something happens on each trial, and we call whatever happens the outcome.
    - These outcomes are *individual* possibilities, like the number we see on top when we roll a die.
- Sometimes we are interested in a combination of outcomes (e.g., a die is rolled and comes up even).
    - A combination of outcomes is called an event.
- When thinking about what happens with combinations of outcomes, things are simplified if the individual trials are independent.
    - Roughly speaking, this means that the outcome of one trial doesn't influence or change the outcome of another.
    - For example, coin flips are independent.

The Law of Large Numbers

- The Law of Large Numbers (LLN) says that the long-run *relative frequency* of repeated independent events gets closer and closer to the *true* relative frequency as the number of trials increases.
    - For example, consider flipping a fair coin many, many times. The overall percentage of heads should settle down to about 50% as the number of outcomes increases.
- The common (mis)understanding of the LLN is that random phenomena are supposed to *compensate* some for whatever happened in the past. This is just not true.
    - For example, when flipping a fair coin, if heads comes up on each of the first 10 flips, what do you think the chance is that tails will come up on the next flip?

Probability

- Thanks to the LLN, we know that relative frequencies settle down in the long run, so we can officially give the name probability to that value.
- Probabilities must be between 0 and 1, inclusive.
    - A probability of 0 indicates impossibility.
    - A probability of 1 indicates certainty.

Equally Likely Outcomes

- When probability was first studied, a group of French mathematicians looked at games of chance in which all the possible outcomes were *equally likely*.
    - It's equally likely to get any one of six outcomes from the roll of a fair die.
    - It's equally likely to get heads or tails from the toss of a fair coin.
- However, keep in mind that events are *not* always equally likely.
    - A skilled basketball player has a better than 50-50 chance of making a free throw.

Personal Probability

- In everyday speech, when we express a degree of uncertainty *without* basing it on long-run relative frequencies, we are stating subjective or personal probabilities.
- Personal probabilities don't display the kind of consistency that we will need probabilities to have, so we'll stick with formally defined probabilities.

Formal Probability

1. Two requirements for a probability:
    - A probability is a number between 0 and 1.
    - For any event **A**, $0 \leq P(\mathbf{A}) \leq 1$.
2. "Something has to happen rule":
    - The probability of the set of all possible outcomes of a trial must be 1.
    - $P(\mathbf{S}) = 1$ (**S** represents the set of all possible outcomes.)
3. Complement Rule:
    - The set of outcomes that are *not* in the event **A** is called the complement of **A**, denoted $\mathbf{A}^{\mathbf{C}}$**.**
    - The probability of an event occurring is 1 minus the probability that it doesn't occur: $P(\mathbf{A}) = 1 - \mathrm{P}(\mathbf{A}^{\mathbf{C}})$
4. Addition Rule:
    - Events that have no outcomes in common (and, thus, cannot occur together) are called disjoint (or mutually exclusive).
    - For two disjoint events **A** and **B**, the probability that one *or* the other occurs is the sum of the probabilities of the two events.
    - $P(\mathbf{A}\ or\ \mathbf{B}) = P(\mathbf{A}) + P(\mathbf{B})$, provided that **A** and **B** are disjoint.
5. Multiplication Rule:
    - For two independent events **A** and **B**, the probability that both **A** and **B** occur is the product of the probabilities of the two events.
    - $P(\mathbf{A}\ and\ \mathbf{B}) = P(\mathbf{A}) \times P(\mathbf{B})$, provided that **A** and **B** are independent.
    - Two independent events **A** and **B** are not disjoint, provided the two events have probabilities greater than zero:
    - Many Statistics methods require an **Independence Assumption**, but *assuming* independence doesn't make it true.
    - Always *Think* about whether that assumption is reasonable before using the Multiplication Rule.

Formal Probability - Notation

- Notation alert:
  - In this text we use the notation $P(\mathbf{A} \text{ or } \mathbf{B})$ and $P(\mathbf{A} \text{ and } \mathbf{B})$.
  - In other situations, you might see the following:
    - $P(\mathbf{A} \cup \mathbf{B})$ instead of $P(\mathbf{A} \text{ or } \mathbf{B})$
    - $P(\mathbf{A} \cap \mathbf{B})$ instead of $P(\mathbf{A} \text{ and } \mathbf{B})$

Putting the Rules to Work

- In most situations where we want to find a probability, we'll use the rules in combination.
- A good thing to remember is that it can be easier to work with the *complement* of the event we're really interested in.

What Can Go Wrong?

- Beware of probabilities that don't add up to 1.
  - To be a legitimate probability distribution, the sum of the probabilities for all possible outcomes must total 1.
- Don't add probabilities of events if they're not disjoint.
  - Events must be disjoint to use the Addition Rule.
- Don't multiply probabilities of events if they're not independent.
  - The multiplication of probabilities of events that are not independent is one of the most common errors people make in dealing with probabilities.
- Don't confuse disjoint and independent—disjoint events *can't* be independent.