SECTION I

Time: 1 hour and 30 minutes

Number of questions: 40

Percentage of total grade: 50

Directions: Solve each of the following problems. Decide which is the best of the choices given and answer in the appropriate place on the answer sheet. No credit will be given for anything written on the exam. Do not spend too much time on any one problem.

1. A poll was conducted in the San Francisco Bay Area after the San Francisco Giants lost the World Series to the Anaheim Angels about whether the team should get rid of a pitcher who lost two games during the series. Five hundred twenty-five adults were interviewed by telephone, and 55% of those responding indicated that the Giants should get rid of the pitcher. It was reported that the survey had a margin of error of 3.5%. Which of the following best describes what is meant by a 3.5% margin of error?

   a. About 3.5% of the respondents were not Giants fans, and their opinions had to be discarded.

   b. It's likely that the true percentage that favor getting rid of the pitcher is between 51.5% and 58.5%.

   c. About 3.5% of those contacted refused to answer the question.

   d. About 3.5% of those contacted said they had no opinion on the matter.

   e. About 3.5% thought their answer was in error and are likely to change their mind.

2. A distribution of SAT Math scores for 130 students at a suburban high school provided the following statistics: Minimum: 485; First Quartile: 502; Median: 520; Third Quartile: 544; Maximum: 610; Mean: 535; Standard Deviation: 88. Define an *outlier* as any score that is at least 1.5 times the interquartile range above or below the quartiles. Which of the following statements is most likely true?

   a. The distribution is skewed to the right and there are no outliers.

   b. The distribution is skewed to the right and there is at least one outlier.

   c. The distribution is skewed to the left and there is at least one outlier.

   d. The distribution is skewed to the left and 65 students scored better than 520.

   e. The distribution is skewed to the right and 65 students scored better than 535.

3. A 2008 ballot initiative in California sought a constitutional ban on same-sex marriage. Suppose a survey prior to the election asked the question, "Do you favor a law that would eliminate the right of same-sex couples to marry?" This question could produce biased results. Which of the following is the most likely reason?

   a. The wording of the question could influence the response.

   b. Same-sex couples are likely to be underrepresented in the sample.

   c. Only those who feel strongly about the issue are likely to respond.

   d. Not all registered voters who respond to the survey are likely to vote.

   e. Married couples are likely to vote the same way.

4. Two plans are being considered for determining resistance to fading of a certain type of paint. Some 1500 of 9500 homes in a large city are known to have been painted with the paint in question. The plans are:

   Plan A:

   a. Random sample 100 homes from all the homes in the city.

   b. Record the amount of fade over a 2-year period.

   c. Generate a confidence interval for the average amount of fade for all 1500 homes with the paint in question.

Plan B:

d. Random sample 100 homes from the 1500 homes with the paint in question.

e. Record the amount of fade over a 2-year period.

f. Generate a confidence interval for the average amount of fade for all 1500 homes with the paint in question.

g. Choose Plan A over Plan B.

h. Either plan is good—the confidence intervals will be the same.

i. Neither plan is good—neither addresses the concerns of the study.

j. Choose Plan B over Plan A.

k. You can't make a choice—there isn't enough information given to evaluate the two plans.

5. Let X be a random variable that follows a t-distribution with a mean of 75 and a standard deviation of 8. Which of the following is (are) equivalent to $P(X > 65)$?

   a. $P(X < 85)$
   b. $P(X \geq 65)$
   c. $1 - P(X < 65)$
   d. I only
   e. II only
   f. III only
   g. I and III only
   h. I, II, and III

6. Which of the following is the best description of a systematic random sample?

   a. A sample chosen in such a way that every possible sample of a given size has an equal chance to be the sample.

   b. After a population is separated into distinct groups, one or more of these groups are randomly selected in their entirety to be the sample.

   c. A value is randomly selected from an ordered list and then every nth value in the list after that first value is selected for the sample.

   d. Select a sample in such a way that the proportion of some variables thought to impact the response is approximately the same in the sample as in the population.

   e. A sample in which respondents volunteer their response.

7. In a famous study from the late 1920s, the Western Electric Company wanted to study the effect of lighting on productivity. They discovered that worker productivity increased with each change of lighting, whether the lighting was increased or decreased. The workers were aware that a study was in progress. What is the most likely cause of this phenomenon? (This effect is known as the Hawthorne Effect.)

   a. Response bias
   b. Absence of a control group
   c. Lack of randomization
   d. Sampling variability
   e. Undercoverage

8. Chris is picked up by the police for stealing hubcaps, but claims that he is innocent, and it is a case of mistaken identity. He goes on trial, and the judge somberly informs the jury that Chris is innocent until proved guilty. That is, they should find him guilty only if there is overwhelming evidence to reject the assumption of innocence. What risk is involved in the jury making a Type-I error?

a. He is guilty, but the jury finds him innocent, and he goes free.

b. He is innocent, and they find him innocent, and he goes free.

c. He is innocent, but the jury finds him guilty, and he goes to jail.

d. He is guilty, and they find him guilty, and he goes to jail.

e. He is guilty, and they correctly reject the assumption of innocence.

9. Given $P(A) = 0.4$, $P(B) = 0.3$, $P(B|A) = 0.2$.
What are $P(A \text{ and } B)$ and $P(A \text{ or } B)$?
   a. $P(A \text{ and } B) = 0.12$, $P(A \text{ or } B) = 0.58$
   b. $P(A \text{ and } B) = 0.08$, $P(A \text{ or } B) = 0.62$
   c. $P(A \text{ and } B) = 0.12$, $P(A \text{ or } B) = 0.62$
   d. $P(A \text{ and } B) = 0.08$, $P(A \text{ or } B) = 0.58$
   e. $P(A \text{ and } B) = 0.08$, $P(A \text{ or } B) = 0.70$

10. A study is to be conducted on a new weatherproofing product for outdoor decks. Four houses with outdoor decks in one suburban neighborhood are selected for the study. Each deck is to be divided into two halves, one half receiving the new product and the other half receiving the product the company currently has on the market. Each of the four decks is divided into North/South sections. Either the new or the old product is randomly assigned to the North side of each of the decks and the other product is assigned to the South side. The major reason for doing this is that

    a. the study is much too small to avoid using randomization.

    b. there are only two treatments being studied.

    c. this controls for known differences in the effect of the sun on the North and South sides of decks.

    d. randomization is a necessary element of any experiment.

    e. this controls for the unknown differential effects of the weather on the North and South sides of decks in this neighborhood.

11. Which of the following best describes a cluster sample of size 20 from a population of size 320?

    a. All 320 names are written on slips of paper and the slips are put into a box. Twenty slips are selected at random from the box.

    b. The 320 names are put into an alphabetical list. One of the first 16 names on the list is selected at random as part of the sample. Every 16th name on the list is then selected for the sample.

    c. The sample will consist of the first 20 people who volunteer to be part of the sample.

    d. Each of the 320 people is assigned a number. Twenty numbers are randomly selected by a computer and the people corresponding to these 20 numbers are the sample.

    e. The 320 names are put into an alphabetical list and the list numbered from 1 to 320. A number between 1 and 304 (inclusive) is selected at random. The person corresponding to that number and the next 19 people on the list are selected for the sample.

12. You are going to conduct an experiment to determine which of four different brands of cat food promotes growth best in kittens ages 4 months to 1 year. You are concerned that the effect might vary by the breed of the cat, so you divide the cats into three different categories by breed. This gives you eight kittens in each category. You randomly assign two of the kittens in each category to one of the four foods. The design of this study is best described as:

    a. randomized block, blocked by breed of cat and type of cat food.

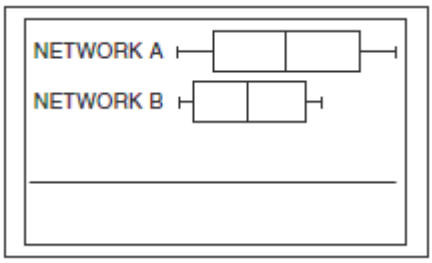
    b. randomized block, blocked by type of cat food.

    c. matched pairs where each two cats are considered a pair.

    d. a controlled design in which the various breed of cats are the controls.

    e. randomized block, blocked by breed of cat.

13.



The boxplots above compare the television ratings for two competing networks. What conclusion(s) can you draw from the boxplots?

a. Network A has more shows than Network B.

b. Network A has a greater range of ratings than Network B.

c. Network A is higher rated than Network B.

d. I and II only

e. II and III only

f. I and III only

g. I, II, and III

h. III only

14. A hypothesis test was used to test $H_0 : \mu = 0.3$ vs. $H_A : \mu \neq 0.3$. The finding was significant for $\alpha = 0.05$ but not for $\alpha = 0.04$. A two-sided confidence interval for μ is constructed. Which of the following is the *smallest* confidence level for which the confidence interval will not contain 0.3?

a. 90%

b. 92%

c. 95%

d. 99%

e. 96%

15. Two months before a statewide election, 532 respondents in a poll of 1000 randomly selected registered voters indicated that they favored Candidate A for governor $(\hat{p}_1 = 0.532)$. One month before the election, a second poll of 900 registered voters was conducted and 444 respondents indicated that they favored Candidate A $(\hat{p}_2 = 0.493)$. A 95% two-proportionz confidence interval for the true difference between the proportions favoring Candidate A in the first and second polls was constructed and found to be (–0.0063, 0.0837). Which of the following is the best interpretation of this interval?

a. There has not been a significant drop in support for Candidate A.

b. There has been a significant drop in support for Candidate A.

c. There has been no change in support for Candidate A.

d. At the 5% level of significance, a test of $H_0 : p_1 = p2$ vs. $H_A : p_1 > p_2$ would yield exactly the same conclusion as the found confidence interval.

e. Since support for Candidate A has fallen below 50%, she is unlikely to win a majority of votes in the general election.

16. A kennel club argues that 50% of dog owners in its area own Golden Retrievers, 40% own Shepherds of one kind or another, and 10% own a variety of other breeds. A random sample of 50 dogs from the area turns up the data in the following table:

GOLDEN RETRIEVER	SHEPHERD	OTHER
27	22	1

What is the value of the X^2 statistic for the goodness-of-fit test on these data?

a. 3.56

b. 2.12

c. 4.31

d. 3.02

e. 2.78

17. A poll is taken to measure the proportion of voters who plan to vote for an ex-actor for Governor. A 95% confidence interval is constructed, based on a sample survey of prospective voters. The conditions needed to construct such an interval are present and the interval constructed is (0.35, 0.42). Which of the following *best* describes how to interpret this interval?

    a. The probability is 0.95 that about 40% of the voters will vote for the ex-actor.

    b. The probability is 0.95 that between 35% and 42% of the population will vote for the ex-actor.

    c. At least 35%, but not more than 42%, of the voters will vote for the ex-actor.

    d. The sample result is likely to be in the interval (0.35, 0.42).

    e. It is likely that the true proportion of voters who will vote for the ex-actor is between 35% and 42%.

18. Two sampling distributions of a sample mean for a random variable are to be constructed. The first (I) has sample size $n_1 = 8$ and the second (II) has sample size $n_2 = 35$. Which of the following statements is not true?

    a. Both sampling distributions I and II will have the same mean.

    b. Distribution I is more variable than Distribution II.

    c. The shape of Distribution I will be similar to the shape of the population from which it was drawn.

    d. The shape of each sampling distribution will be approximately normal.

    e. The shape of Distribution II will be approximately normal.

19. A researcher wants to determine if a newly developed anti-smoking program can be successful. At the beginning of the program, a sample of 1800 people who smoked at least 10 cigarettes a day were recruited for the study. These volunteers were randomly divided into two groups of 900 people. Each group received a set of anti-smoking materials and a lecture from a doctor and a cancer patient about the dangers of smoking. In addition, the treatment group received materials from the newly developed program. At the end of 2 months, 252 of the 900 people in the control group (the group that did not receive the new materials) reported that they no longer smoked. Out of the 900 people in the treatment group, 283 reported that they no longer smoked. Which of the following is an appropriate conclusion from this study?

    a. Because the *P*-value of this test is greater than $\alpha = 0.05$, we cannot conclude that the newly developed program is significantly different from the control program at reducing the rate of smoking.

    b. Since the proportion of people who have quit smoking in the experimental group is greater than in the control group, we can conclude that the new program is effective at reducing the rate of smoking.

    c. Because the *P*-value of this test is less than $\alpha = 0.05$, we can conclude that the newly developed program is significantly different from the control program at reducing the rate of smoking.

    d. Because the difference in the proportions of those who have quit smoking in the control group (28%) and the experimental group (31.4%) is so small, we cannot conclude that there is a statistically significant difference between the two groups in terms of their rates of quitting smoking.

e. The standard deviation of the difference between the two sample proportions is about 0.022. This is so small as to give us good evidence that the new program is more effective at reducing the rate of smoking.

Questions 20 and 21 refer to the following information:

At a local community college, 90% of students take English. 80% of those who don't take English take art courses, while only 50% of those who do take English take art.

20. What is the probability that a student takes art?
    a. 0.80
    b. 0.53
    c. 0.50
    d. 1.3
    e. 0.45
21. What is the probability that a student who takes art doesn't take English?
    a. 0.08
    b. 0.10
    c. 0.8
    d. 0.85
    e. 0.15
22. Which of the following is the best reason to use a *t*-distribution rather than a normal distribution when testing for a population mean?
    a. You should always use a *t*-distribution for small samples.
    b. You are unable to compute the sample standard deviation.
    c. The normal distribution is too variable.
    d. The population standard deviation is unknown.
    e. *t*-distributions are very similar to the normal distribution for large samples.
23. A study of 15 people ages 5 through 77 was conducted to determine the amount of leisure time people of various ages have. The results are shown in the following computer printout:

Time = 7.85 + 0.0094 Age

Predictor	Coef	St Dev	*t* ratio	*P*
Constant	7.845	3.032	2.59	.023
Age	0.00935	0.07015	0.13	.896

s = 5.628 R-sq = 0.1% R-sq(adj) = 0.0%

Which of the following is the 99% confidence interval for the true slope of the regression line?

    a. 0.00935 ± 3.012(0.07015)
    b. 0.00935 ± 2.977(5.628)
    c. 7.845 ± 3.012(0.07015)
    d. 0.00935 ± 2.977(0.07015)
    e. 0.00935 ± 3.012(5.628)

24. You want to conduct a survey to determine the types of exercise equipment most used by people at your health club. You plan to base your results on a random sample of 40 members. Which of the following methods will generate a random simple random sample of 40 of the members?

    a. Mail out surveys to every member and use the first 40 that are returned as your sample.

    b. Randomly pick a morning and survey the first 40 people who come in the door that day.

    c. Divide the number of members by 40 to get a value *k*. Choose one of the first *k*th names on the list using a random number generator. Then choose every *k*th name on the list after that name.

    d. Put each member's name on a slip of paper and randomly select 40 slips.

    e. Get the sign-in lists for each day of the week, Monday through Friday. Randomly choose 8 names from each day for the survey.

25. The following numbers are given in ascending order: 3, 4, *x*, *x*, 9, *w*, 13, 28, *y*, *z*. Which of the following gives a five-number summary of the data?

    a. $\{3, x, \frac{w+9}{2}, 28, z\}$

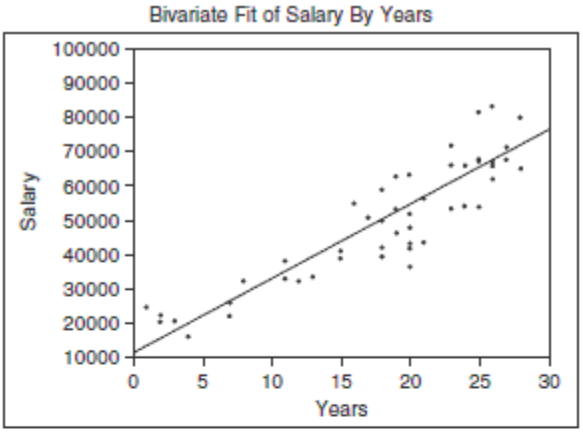
    b. $\{3, \frac{x+4}{2}, \frac{w+9}{2}, 28, z\}$

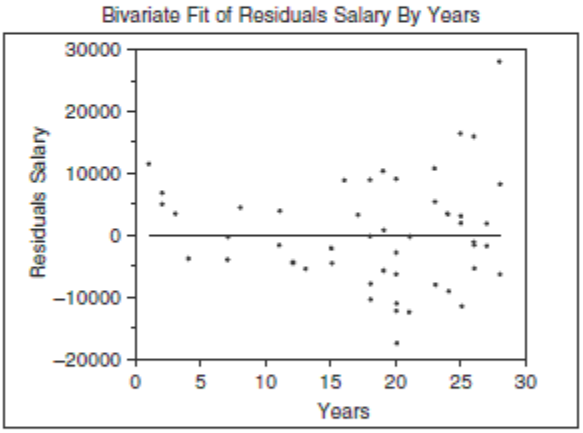
    c. {3, 4, 9, 13, 28]

    d. {3, *x*, *w*, 28, *z*}

    e. There isn't enough information to identify all five numbers in the five-number summary.

26. The salaries and years of experience for 50 social workers was collected and a regression analysis was conducted to investigate the nature of the relationship between the two variables. R-sq. = 0.79. The results are as follows:





Linear Fit
Salary = 11369.416 + 2141.3092*Years

Parameter Estimates

TERM	ESTIMATE	STD ERROR	t RATIO	PROB > |t|
Intercept	11369.416	3160.249	3.60	0.0008
Years	2141.3092	160.8358	13.31	<.0001

Which of the following statements is *least* correct?

a. There is a statistically significant predictive linear relationship between Years of Experience and Salary.

b. The residual plot indicates that a line is a good model for the data for all years.

c. There appears to be an outlier in the data at about 28 years of experience.

d. The variability of salaries increases as years of experience increases.

e. For each additional year of experience, salary is predicted to increase by about $2141.

27. A wine maker advertises that the mean alcohol content of the wine produced by his winery is 11%. A 95% confidence interval, based on a random sample of 100 bottles of wine yields a confidence interval for the true alcohol content of (10.5, 10.9) Could this interval be used as part of a hypothesis test of the null hypothesis H_0: $p = 0.11$ versus the alternative hypothesis H_A: $p \neq 0.11$ at the 0.05 level or confidence?

a. No, you cannot use a confidence interval in a hypothesis test.

b. Yes, because 0.11 is not contained in the 95% confidence interval, a two-sided test at the 0.05 level of significance would provide good evidence that the true mean content is different from 11%.

c. No, because we do not know that the distribution is approximately normally distributed.

d. Yes, because 0.11 is not contained in the 95% confidence interval, a two-sided test at the 0.05 level of significance would fail to reject the null hypothesis.

e. No, confidence intervals can only be used in one-sided significance tests.

28. Tom's career batting average is 0.265 with a standard deviation of 0.035. Larry's career batting average is 0.283 with a standard deviation of 0.029. The distribution of both averages is approximately normal. They play for different teams and there is reason to believe that their career averages are independent of each other. For any given year, what is the probability that Tom will have a higher batting average than Larry?

a. 0.389

b. 0.345

c. 0.589

d. 0.655

e. You cannot answer this question since the distribution for the difference between their averages cannot be determined from the data given.

29. An advice columnist asks readers to write in about how happy they are in their marriages. The results indicate that 79% of those responding would not marry the same partner if they had it to do all over again. Which of the following statements is most correct?

a. It's likely that this result is an accurate reflection of the population.

b. It's likely that this result is higher than the true population proportion because persons unhappy in their marriages are most likely to respond.

c. It's likely that this result is lower than the true population proportion because persons unhappy in their marriages are unlikely to respond.

d. It's likely that the results are not accurate because people tend to lie in voluntary response surveys.

e. There is really no way of predicting whether the results are biased or not.

30. A national polling organization wishes to generate a 98% confidence interval for the proportion of voters who will vote for candidate lam Sleazy in the next election. The poll is to have a margin of error of no more than 3%. What is the minimum sample size needed for this interval?

a. 6032

b. 1508

c. 39

d. 6033

e. 1509

31. In a test of the hypothesis H_0: $p = 0.7$ against H_A: $p > 0.7$ the power of the test when $p = 0.8$ would be greatest for which of the following?

a. $n = 30$, $\alpha = 0.10$

b. $n = 30$, $\alpha = 0.05$

c. $n = 25$, $\alpha = 0.10$

d. $n = 25$, $\alpha = 0.05$

e. It cannot be determined from the information given.

32. A research team is interested in determining the extent to which food markets differ in prices for store-brand items and the same name brand items. They identify a "shopping basket" of 10 items for which they know store-brand and name-brand items exist (e.g, peanut

butter, canned milk, grape juice, etc.). In order to control market-to-market variability, they decide to conduct the study only at one major market chain and will select just one market in each of twelve geographically diverse cities. For each market selected they will compute the mean for the 10 store-brand items (call it $\overline{x}_S$) and also for the 10 name-brand items ($\overline{x}_N$). They then intend to conduct a two-sample *t*-test ($H_0 : \mu_N - \mu_S = 0$ vs. $H_A : \mu_N - \mu_S > 0$) in order to determine if there is a statistically significant difference between the average prices of the two types of items. This procedure is not appropriate because

a. the sample sizes are too small to use a twosample test.

b. the variances are most likely not the same.

c. there is no randomization of treatments.

d. the samples are not independent.

e. they should be using a two-sample *z*-test.

33. A researcher was interested in determining the relationship between pulse rate (in beats/minute) and the time (in minutes) it took to swim a fixed distance. Based on 25 trials in the pool, the correlation coefficient between time and pulse rate was found to be –0.654 (that is, large times—going slowly—were associated with slower pulses). Prior to publication, the researcher decided to change the time measurements to seconds (each of the 25 times was multiplied by 60). What would this conversion do to the correlation between the two variables?

a. Since the units on only one of the variables was changed, the correlation between the two variables would decrease.

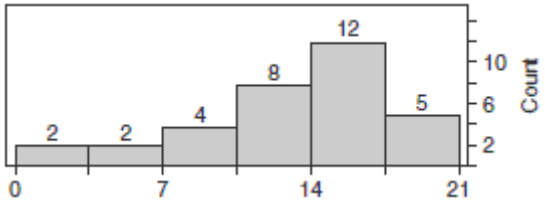
b. The correlation would change proportional to the change in the units for time.

c. The correlation between the two variables would change, but there is no way, based on the information given, to know by how much.

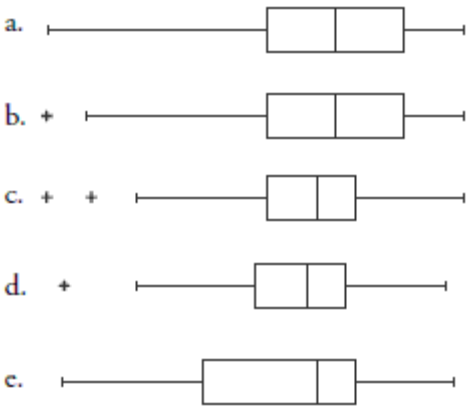
d. Changing the units of measurement has no effect on the correlation coefficient. Hence, the correlation would be the same.

e. Since changing from minutes to seconds would result in larger times, the correlation would actually increase.

34. The following histogram displays the scores of 33 students on a 20-point Introduction to Statistics quiz. The lowest score, 0, is an outlier. The next lowest score, 2, is not an outlier.

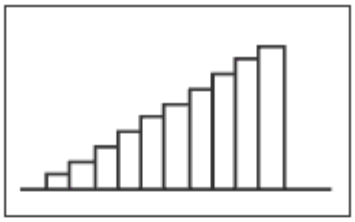


Which of the following boxplots best represents the data shown in the histogram?

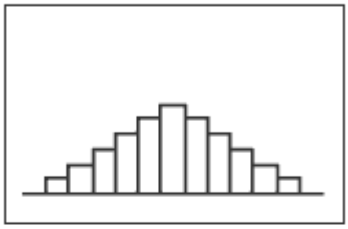


35. For which one of the following distributions is the mean most likely to be less than the median?

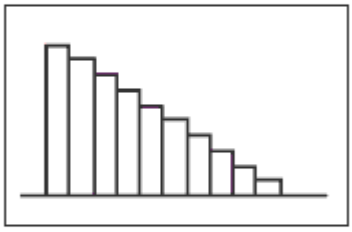
a.



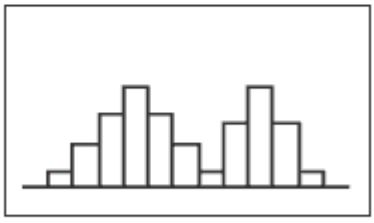
b.



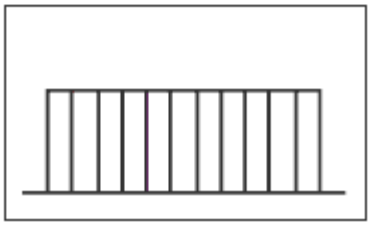
c.



d.



e.



36. An SAT test preparation program advertises that its program will improve scores on the SAT test by at least 30 points. Twelve students who have not yet taken the SAT were selected for the study and were administered the test. The 12 students then went through the 3-week testprep course. The results of the testing were as follows:

Student	1	2	3	4	5	6	7	8	9	10	11	12
Before	475	500	499	477	540	608	510	425	495	502	530	487
After	495	540	495	522	555	684	535	460	522	529	560	512

Assuming that the conditions necessary to conduct the test are present, which of the following significance tests should be used to determine if the test-prep course is effective in raising score by the amount claimed?

a. A two-sample *t*-test
b. A chi-square test of independence
c. A one-sample *t*-test
d. A *t*-test for the slope of a regression line
e. A two-sample *z*-test

37. Which of the following statements is (are) correct?

a. The area under a probability density curve for a continuous random variable is 1.
b. A random variable is a numerical outcome of a random event.
c. The sum of the probabilities for a discrete random variable is 1.
d. II only
e. I and II
f. I and III
g. II and III
h. I, II, and III

38. Let *X* be the number of points awarded for winning a game that has the following probability distribution:

X	0	2	3
P(*X*)	0.2	0.5	0.3

Let *Y* be the random variable whose sum is the number of points that results from two independent repetitions of the game. Which of the following is the probability distribution for *Y* ?

a.

Y	0	2	3
P(*Y*)	0.2	0.5	0.3

b.

Y	0	4	6
P(*Y*)	0.2	0.5	0.3

c.

Y	0	2	3	4	6
P(*Y*)	0.2	0.25	0.15	0.25	0.15

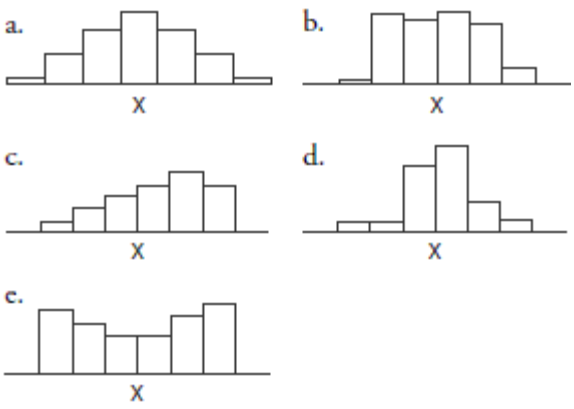
d.

Y	0	2	3	4	5	6
P(*Y*)	0.04	0.2	0.12	0.25	0.3	0.09

e.

Y	0	2	3	4	5	6
P(*Y*)	0.04	0.10	0.06	0.25	0.15	0.09

39. Each of the following histograms represents a simulation of a sampling distribution for an estimator of a population parameter. The true value of the parameter is *X*, as shown on the scale. The domain of possible outcomes is the same for each estimator and the frequency axes (not shown) are the same. Which histogram represents the best estimator of *X*?



40. A weight-loss clinic claims an average weight loss over 3 months of at least 15 pounds. A random sample of 50 of the clinic's patrons shows a mean weight loss of 14 pounds with a standard deviation of 2.8 pounds. Assuming the distribution of weight losses is approximately normally distributed, what is the most appropriate test for this situation, the value of the test statistic, and the associated *P*-value?

    a. *z*-test; $z = -2.53$; *P*-value = 0.0057
    b. *t*-test; $t = -2.53$; $0.01 < P\text{-value} < 0.02$
    c. *z*-test; $z = 2.53$; *P*-value = 0.0057
    d. *t*-test; $t = -2.53$; $0.005 < P\text{-value} < 0.01$
    e. *z*-test; $z = 2.53$; *P*-value = 0.9943

SECTION II

Time: 1 hour and 30 minutes

Number of problems: 6

Percentage of total grade: 50

General Instructions

There are two parts to this section of the examination. Part A consists of five equally weighted problems that represent 75% of the total weight of this section. Spend about 65 minutes on this part of the exam. Part B consists of one longer problem that represents 25% of the total weight of this section. Spend about 25 minutes on this part of the exam. You are not necessarily expected to complete all parts of every question. Statistical tables and formulas are provided.

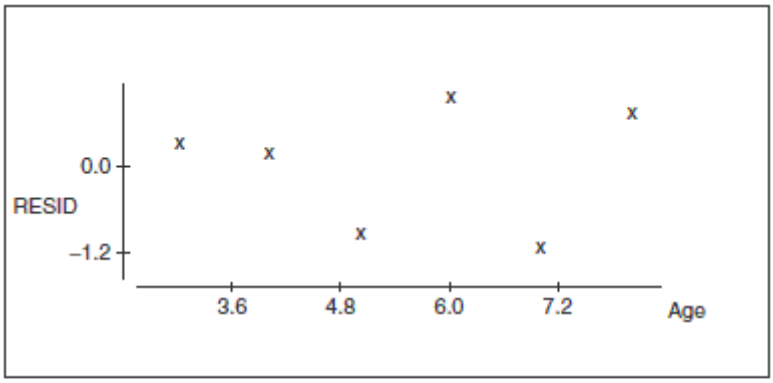
- Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as the accuracy of your final answers. Correct answers without support work may not receive credit.

Statistics, Section II, Part A, Questions 1–5

Spend about 65 minutes on this part of the exam; percentage of Section II grade: 75.

Directions: Show all your work. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

1. David was comparing the number of vocabulary words children know about transportation at various ages. He fit a least-squares regression line to the data. The residual plot and part of the computer output for the regression are given below.



Predictor	Coef	St Dev	*t* ratio	*P*
Constant	3.371	1.337	2.52	.065
Age	2.1143	0.2321	9.11	.001

s = 0.9710 R-sq = 95.4% R-sq(adj) = 94.3%

a. Is a line an appropriate model for these data? Explain.

b. What is the equation of the least-square regression line for predicting the number of words from age?

c. What is the predicted number of words for a child of 7.5 years of age?

d. Interpret the slope of the regression line in the context of the problem.

e. Would it be appropriate to use the model to predict the number of words a 12-yearold would know?

2. Students at Dot.Com Tech are allowed to sign up for one math class each semester. The numbers in each grade level signing up for various classes for next semester are given in the following table.

	GEOMETRY	ALGEBRA II	ANALYSIS	CALCULUS AB	TOTAL
10th Grade	125	74	23	3	**225**
11th Grade	41	92	72	25	**230**
12th Grade	12	47	99	62	**220**
Total	178	213	194	90	**675**

a. What is the probability that a student will take calculus?

b. What is the probability that a 12th grader will take either analysis or calculus?

c. What is the probability that a person taking algebra II is a 10th grader?

d. Consider the events, "A student takes geometry" and "A student is a 10th grader."

Are these events independent? Justify your answer.

3. The state in which you reside is undergoing a significant budget crisis that will affect education. Your school is trying to decide how many sections of upper-level mathematics classes to offer next year. It is very expensive to offer sections that aren't full, so the school doesn't want to offer any more sections than it absolutely needs to. The assistant principal in charge of scheduling selects a random sample of 60 current sophomores and juniors. Fifty-five of them return the survey, and 48 indicate that they intend to take math during the coming year. If 80% or more of the students actually sign up for math, the school will need to add a section.

a. On the basis of the survey data, would you recommend to the assistant principal that an additional class of upper division mathematics should be scheduled? Give appropriate statistical evidence to support your recommendation.

b. Five of the 60 who received surveys failed to return them. If they had returned them, how might it have affected the assistant principal's decision? Explain.

4. It is known that the symptoms of adult depression can be treated effectively with either therapy, antidepressants, or a combination of the two. A pharmaceutical company wants to test a new antidepressant against an older medication that has been on the market for several years. One hundred fifty volunteers who have been diagnosed with depression, and who have not been taking any medication for it, are available for the study. This group contains 72 men and 78 women. Sixty of the volunteers have been in therapy for their depression for at least 3 months.

a. Design a completely randomized experiment to test the new medication. Include a brief explanation of the randomization process.

b. Could the experiment you designed in part (a) be improved by blocking? If so, design an improved study that involves blocking. If not, explain why not.

5. The 1970 draft lottery was suspected to be biased toward birthdays later in the year. Because there are 366 possible birthdays, in a fair drawing we would expect to find, each month, an equal number of selections less than or equal to 183 and greater than or equal to 184. The following table shows the data from the 1970 draft lottery.

	NUMBER SELECTED ≤ 183	NUMBER SELECTED ≥ 184	TOTALS
January	12	19	31
February	12	17	29
March	10	21	31
April	11	19	30
May	14	17	31
June	14	16	30
July	14	17	31
August	19	12	31
September	17	13	30
October	13	18	31
November	21	9	30
December	26	5	31
Totals	**183**	**183**	**366**

Do these data give evidence that the 1970 draft lottery was not fair? Give appropriate statistical evidence to support your conclusion.

Statistics, Section II, Part B, Question 6

Spend about 25 minutes on this part of the exam; percentage of Section II grade: 25.

Directions: Show all of your work. Indicate clearly the methods you use because you will be graded on the correctness of your methods as well as on the accuracy of your results and explanation.

6. A lake in the Midwest has a restriction on the size of trout caught in the lake. The average length of trout over the years has been 11 inches with a standard deviation of 0.6 inches. The lengths are approximately normally distributed. Because of overfishing during the past few years, any fish under 11.5 inches in length must be released.

    a. What is the probability that a fisherman will get lucky and catches a fish she can keep? Round your answer to the nearest tenth.

    b. Design a simulation to determine the probability how many fish the fisherman must catch, on average, in order to be able to take home five trout for dinner.

    c. Use the table of random digits between parts (d) and (e) of this problem to conduct five trials of your simulation. Show your work directly on the table.

    d. Based on your simulation, what is your estimate of the average number of fish that need to be caught in order to catch five she can keep?

79692	51707	73274	12548	91497	11135	81218	79572	06484	87440
41957	21607	51248	54772	19481	90392	35268	36234	90244	02146
07094	31750	69426	62510	90127	43365	61167	53938	03694	76923
59365	43671	12704	87941	51620	45102	22785	07729	40985	92589

    e. What is the theoretical expected number of fish that need to be caught in order to be able to keep five of them?

Solutions to Practice Exam 1, Section I

1. The correct answer is (b). The confidence level isn't mentioned in the problem, but polls often use 95%. If that is the case, we are 95% confident that the true value is within 3.5% of the sample value.

2. The correct answer is (b). Since the mean is noticeably greater than the median, the distribution is likely skewed to the right. Another indication of this is the long "whisker" on a boxplot of the five-number summary. IQR = 544 – 502 = 42. An outlier is any value less than 502 – 1.5(542) = 439 or greater than 544 + 1.5(42) = 607. Since the maximum value given (610) is greater than 607, there is at least one outlier.

3. The correct answer is (a). The most likely bias would be to influence people to oppose such a law since many voters are resistant to constitutional amendments restricting rights. Compare this to the question, "Do you favor a law that would provide that only marriage between a man and a woman is valid or recognized in California?" which could influence voters to favor the amendment. This was, in fact, a legal issue in California prior to the election. The original title of the amendment was "Limit on Marriage." The Attorney General of California, Jerry Brown, changed the title to include the phrase "eliminates (the) right of same-sex couples to marry." When challenged in court by proponents of the amendment, the title change was upheld based on the fact that the right of same-sex couples to marry had been given full legal status by the state supreme court earlier in the year.

4. The correct answer is (d). Choosing your sample from only the homes in your population of interest gives you a larger sample on which to base your confidence interval. If you use Plan A, you will end up with many homes painted with different paint than the paint of interest.

5. The correct answer is (e). The *t*-distributions are symmetric about their means. I is the mirror image of $P(X > 65)$. II, in a continuous distribution, is equivalent to $P(X > 65)$—this would not be true in a discrete distribution (e.g., a binomial).

6. The correct answer is (c). (a) describes a simple random sample. (b) describes a cluster sample. (d) describes stratified random sampling. (e) describes a voluntary response sample.

7. The correct answer is (a). There is a natural tendency on the part of a subject in an experiment to want to please the researcher. It is likely that the employees were increasing their production because they wanted to behave in the way they thought they were expected to.

8. The correct answer is (c). A Type-I error occurs when a true hypothesis is incorrectly rejected. In this case, that means that the assumption of innocence is rejected, and he is found guilty.

9. The correct answer is (b). $P(\text{A and B}) = P(\text{A}) \cdot P(\text{A}|\text{B}) = (0.4)(0.2) = 0.08$. $P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B}) = 0.4 + 0.3 - 0.08 = 0.62$.

10. The correct answer is (e). The purpose of randomization is to control for the unknown effects of variables that might affect the response, in this case the differential effects of North or South placement. (a) is incorrect since studies of any size benefit from randomization. (b) is simply nonsense—the number of treatments does not affect the need to randomize. (c) involves blocking and would be correct if we know in advance that there were differential effects based on a North/South placement—but nothing in the problem indicates this. (d) is not incorrect, but it's not the reason we are randomizing in this situation.

11. The correct answer is (e). By definition a "cluster sample" occurs when a population is divided into groups and then a group or groups is randomly selected. (a) is a simple random sample; (b) is a systematic sample; (c) is a self-selected sample and is not random; (d) is asimple random sample.

12. The correct answer is (e). Be clear on the difference between the treatment (type of cat food in this problem) and the blocking variable (breed of cat).

13. The correct answer is (b). The box is longer for Network A and the ends of the whiskers are further apart than Network B $\Rightarrow$ Network A has a greater range of ratings than Network B. The 3rd quartile, the median, and the 1st quartile of Network A are higher than Network B, which can be interpreted to mean that Network A is higher rated than Network B. I is not correct because there is no way to tell how many values are in a boxplot.between the treatment (type of cat food in this problem) and the blocking variable (breed of cat).

14. The correct answer is (e). If a significance test rejects H_0 at $\alpha = 0.05$, then a two-sided 95% confidence interval *will* not contain 0.3. If the finding is not significant at $\alpha = 0.04$, then a two-sided 96% confidence interval *will* contain 0.3. Hence, any confidence level 95% or less will contain 0.3 and any confidence level 96% or higher will not.

15. The correct answer is (a). Since 0 is in the interval, it is possible that the true difference between the proportions is 0, i.e, that there has not been a significant drop in support. Note that a 90% interval, but not a 95% interval, would contain only positive numbers and would provide statistically significant evidence of a drop in support. (d) is false since it's a one-sided test (with a *P*-value of 0.046) and the

confidence interval is two sided. A two-sided test ($H_A : P_1 \neq p_2$) would yield the same conclusion ($P = 0.092$) as the confidence interval. (b), (c), and (d) are simply wrong.

16. The correct answer is (a). The expected values are: 0.5 × 50 = 25 Golden Retrievers, 0.8 × 50 = 20 Shepherds; and 0.1 × 50 = 5 Others.

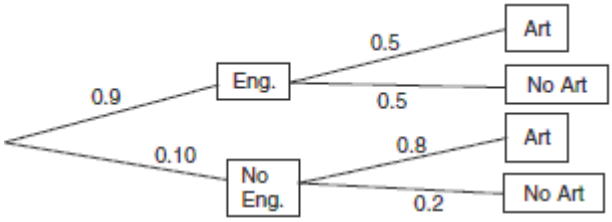
$$X^2 = \frac{(27-25)^2}{25} + \frac{(22-20)^2}{20} + \frac{(1-5)^2}{5} = 3.56$$

17. The answer is (e). We are 95% confident that the true proportion who will vote for the former actor is in the interval (0.35, 0.42). This means that the true proportion is likely to be in this interval.

18. The correct answer is (d). While the central limit theorem argues that the shape of Distribution II will be approximately normal, the sample size for Distribution I is too small for the CLT to apply. The best we can say is that the distribution of a sampling distribution for a small sample will be similar to the original population (hence, (c) is true). Since we are not given the shape of the original population, we cannot make the claim that the distribution for the smaller sample size will be approximately normal.

19. The correct answer is (a). The *P*-value for the two-proportion *z*-test is 0.055. This isn't much over 0.05, but it is enough to say that we do not have, at the 0.05 level, a statistically significant difference between the two findings. Note that (a) and (c) are mutually exclusive. If (a) is correct, which it is, then (c) must be false. (b) is not correct since it does not take into account random variation. While the conclusion in (d) is correct, the statement is not—the raw difference between two values is not what allows us to make conclusions about statistical differences between groups. (e), while it does get at the variability between the sample proportions, doesn't tell us anything by itself.

20. The correct answer is (b). The following tree diagram illustrates the situation:



$$P(\text{Art} \mid \text{Eng}) = (0.9)(0.5) = 0.45$$
$$P(\text{No Art} \mid \text{Eng}) = (0.9)(0.5) = 0.45$$
$$P(\text{Art} \mid \text{Eng}) = (0.1)(0.8) = 0.08$$
$$P(\text{No Art} \mid \text{Eng}) = (0.1)(0.2) = 0.02$$

Now, *P* (student takes art) = 0.45 + 0.08 = 0.53.

21. The correct answer is (e). *P* (doesn't take English | does take art)

$$= \frac{0.08}{0.45 + 0.08} = 0.15$$

22. The correct answer is (d). In general, when testing for a population mean, you should use a *t*-distribution unless the population standard deviation is known—which it rarely is in practice. (a), (b), and (c) are simply incorrect. (e) is a correct statement but is not the reason you would use *t* rather than *z* (in fact, if it argues anything, it argues that there is no practical numerical difference between using t or z for large samples).

23. The correct answer is (a). df = $n - 2 = 15 - 2 = 13 \Rightarrow t^* = 3.012$ (if you have a TI-84 with the invT function, invT (0.995, 13) = 3.0123). The standard error of the slope of the regression line (found under "St Dev" after "Age"), sb, is 0.07015. The confidence interval, therefore, is 0.00935 ± 3.012(0.07015). (Newer TI-84's have a LinRegTInt in the STAT TESTS menu. However, that is of no help here since the calculator requires that the data be in lists—there is no option to enter summary statistics.)

24. The correct answer is (d). To be a simple random sample, every possible sample of size 40 must be equally likely. Only (d) meets this standard. Note that (c) and (e) are perfectly valid ways of collecting a random sample. At the start, each member of the population has an equal chance to be in the sample. But they are not SRS's.

25. The correct answer is (a). First find the median. Since there are 10 terms, the median is the mean of the middle two terms: $\left(\frac{w+9}{2}\right)$. The first quartile is the median of the five terms less than the median (x) and the third quartile is the median of the five terms greater than the median (28). The minimum value is 3 and the maximum is z.

26. The correct answer is (b). Note that the variability of the residuals increases as years of experience increases so that the pattern for all years is not truly random. (a) is correct based on the t-test for the slope of the regression line given as "<.0001." (c) is correct as the residual of that point is very large compared to most of the points. (d) is correct and is the reason that (b) is not a true statement. (e) is the standard interpretation of the slope of a regression line.

27. The correct answer is (b). A confidence interval can be used in place of a significance test in a hypothesis test (for a population mean or the difference between two population means) with a two-sided alternative. In this case, the evidence supports the alternative.

28. The correct answer is (b). Since the distributions are independent and approximately normal, we use $\mu_{X-Y} = \mu_{X-Y}$ and $\sigma_{X-Y} = \sqrt{\sigma_X^2 + \sigma_Y^2}$. Tom ($T$) has N(0.265, 0.035) and Larry (L) has N(0.283, 0.029). Hence, $\mu_{T-L} = 0.265–0.283 = –0.018$ and $\sigma_{T-L} = \sqrt{0.035^2 + 0.029^2} = 0.045$. $T – L$ then has the distribution N(–0.018, 0.045). We need to know the probability that $T – L$ is positive (since we require that T be greater than L) in this distribution. $P(T – L > 0)$ $= P\left(z > \frac{0-(-0.018)}{0.045} = 0.4\right) = 0.345$. On the TI-83/84 calculator, normal cdf(0.4, 0.045)= 0.3446. Also, normal cdf (0,100,0.018,0.045)=0.3446.

29. The correct answer is (b). The tendency in voluntary response surveys is for people who feel most strongly about an issue to respond. If people are happy in their marriage, they are less likely to respond.

30. The correct answer is (e). The upper critical z for a 98% confidence interval is $z^* = 2.33$ (from Table A; on the TI-83/84, invNorm (0.99)= 2.326). In the expression $n \geq \left(\frac{z^*}{M}\right)^2 P^*(1-P^*)$, we choose $P^* = 0.5$ since we are not given any reason to choose something else. In this case the "recipe" becomes $n \geq \left(\frac{z^*}{2M}\right)^2$. The sample size needed is $n \geq \left(\frac{2.33}{2(0.03)}\right)^2 = 1508.03$. Choose $n = 1509$. (Note: If you use $z^* = 2.326$ rather than 2.33, you will get $n \geq 1502.9 \Rightarrow$ choose n = 1503.)

31. The correct answer is (a). Power can be increased by increasing n, increasing α moving the alternative further away from the null, reducing the variability. This choice provides the best combination of large n and large α.

32. The correct answer is (d). The data are paired in that two measurements are being taken at each of 12 different stores. The correct analysis would involve a one-sample t-test ($H_0 : \mu_d = 0$ vs. $H_A : \mu_d > 0$ where $d = \overline{x}_N - \overline{x}_S$ for each of the 12 pairings).

33. The correct answer is (d). The correlation coefficient is not affected by any linear transformation of the variables. Changing the units of measurement is a linear transformation.

34. The correct answer is (b). The key is to note that there is an outlier, which eliminates (a) and (e), and only one outlier, which eliminates (c). The histogram is skewed to the left, which shows in the boxplot for (b) but not for (d) which is, except for the outlier, more symmetric.

35. The correct answer is (a). The mean is pulled in the direction of skewness.

36. The correct answer is (c). The data are paired, which means that we are testing $H_0 : \mu_d = 30$ vs. $H_A : \mu > 30$, where d = Before – After for each student. That is, there is only one sample.

37. The correct answer is (e).

38. The correct answer is (d). The computations are shown in the following table:

Y	0	2	3	4	5	6
How:	0, 0	0, 2 or 2, 0	0,3 or 3, 0	2, 2	2, 3 or 3, 2	3, 3
P(Y)	(0.2)(0.2) = 0.04	2(0.2)(0.5) = 0.2	2(0.2)(0.3) = 0.12	(0.5)(0.5) = 0.25	2(0.5)(0.3) = 0.3	(0.3)(0.3) = 0.09

39. The correct answer is (d). While all of the graphs tend to center around X, the true value of the parameter, (d) has the least variability about X. Since all of the graphs have roughly the same low bias, the best estimator will be the one with the least variability.
40. The correct answer is (d). We are told that the distribution of weight losses is approximately normal, but we are not given the population standard deviation. Hence the most appropriate test is a t-test. Now, $t = \frac{14-15}{2.8/\sqrt{50}} = -2.53$, df = 50 – 1 = 49 $\Rightarrow$ 0.005 < P-value < 0.01 (from Table B, rounding down to df = 40; using the TI-83/84, we have tcdf(–100,–2.53, 49)= 0.0073). Note that the P-value for the z-test in (a) is quite close. However, a z-test is not the *most* appropriate test since we do not know the population standard deviation.

Solutions to Practice Exam 1, Section II, Part A

Solution to #1

1. We note that $r^2 = 0.954 \Rightarrow r = 0.98$, so there is a strong linear correlation between the variables. We know that the correlation is positive since the slope of the regression line is positive. Also, the residual plot shows no obvious pattern, indicating that the line is a good model for the data.
2. *Words* = 3.371 + 2.1143(*Age*).
3. *Words* = 3.371 + 2.1143(7.5) = 19.2.
4. For each year a child grows, the number of words he or she knows about transportation is predicted to grow by 2.1.
5. No. This would require extrapolating beyond the range of the data.

Solution to #2

1. P(a student takes calculus) = 90/675 = 0.133.
2. P(a student takes analysis or calculus given that he/she is in the 12th grade)
$$= \frac{99+62}{220} = \frac{161}{220} = 0.732.$$
3. P(a student is in the 10th grade given that he/she is taking algebra II)
$$= \frac{74}{213} = 0.347.$$
4. Let A = "A student takes geometry" and B = "A student is a 10th grader." A and B are independent events if and only if $P(A) = P(A \mid B)$.
$$P(A) = \frac{178}{675} = 0.264.$$
$$P(A|B) = \frac{125}{225} = 0.556.$$

Thus, the events are not independent.

Solution to #3

1. Let p = the true proportion of students who will sign up for upper division mathematics classes during the coming year.

$$H_0 : p \le 0.80.$$
$$H_A : p > 0.80.$$

We want to use a one-proportion z-test, at the 0.05 level of significance. We note that we are given that the sample was a random sample, and that $np = 55(0.8) = 44$ and $55(1 - 0.8) = 11$ are both larger than 5 (or 10). Thus the conditions needed for this test are present.

$$\hat{p} = \frac{48}{55} = z = \frac{0.873 - .080}{\sqrt{\frac{0.8(1-0.8)}{55}}} = 1.35 \Rightarrow P\text{-value} = 1 - 0.9115 = 0.0885$$

(from Table A; on the TI-83/84, the P-value = normalcdf(1.35,100)). Since $P > \alpha$, we cannot reject the null hypothesis. The survey evidence is not strong enough to justify adding another section of upper-division mathematics.

2. If all five of the other students returned their surveys, there are two worst-case scenarios: all five say they will sign up; all five say they will not sign up. If all five say they will not sign up, then an even lower percentage say they need the class (48/60 rather than 48/55) and our decision not to offer another class would not change. If all five say they will sign up, then

$$\hat{p} = \frac{53}{60} = 0.883 \rightarrow z = \frac{0.883 - 0.080}{\sqrt{\frac{0.8(1-0.8)}{60}}} = 1.61 \Rightarrow P\text{-value} = 1 - 0.9463 = 0.0537$$

At the 5% level of significance, this is still not quite enough to reject the null. However, it's very close, and the assistant principal might want to generate some additional data before making a final decision.

Solution to #4

1. Randomly divide your 150 volunteers into two groups. One way to do this would be to put each volunteer's name on a slip of paper, put the papers into a box, and begin drawing them out. The first 75 names selected would be in group A, and the other 75 would be in group B. Alternatively, you could flip a coin for each volunteer. If it came up heads, the volunteer goes into group A; if tails, the volunteer goes into group B. The second method would likely result in unequal size groups.

   Administer one group the new medication (treatment group), and administer the old medication to the other group (control) for a period of time. After enough time has passed, have each volunteer evaluated for reduction in the symptoms of depression. Compare the two groups.

2. Because we know that being in therapy can affect the symptoms of depression, block by having the 60 people who have been in therapy be in one block and the 90 who have not been in therapy be in the other block. Then, within each block, conduct an experiment as described in part (a).

Solution to #5

H_0: Month of birth and draft number are independent.
H_A: Month of birth and draft number are not independent.

This is a two-way table with 12 rows and 2 columns. A chi-square test of independence is appropriate. The numbers of expected values are given in the table below:

	EXPECTED NUMBER SELECTED ≤ 183	EXPECTED NUMBER SELECTED ≥ 184
January	15.5	15.5
February	14.5	14.5
March	15.5	15.5
April	15	15
May	15.5	15.5
June	15	15
July	15.5	15.5
August	15.5	15.5
September	15	15
October	15.5	15.5
November	15	15
December	15.5	15.5

Because all expected values are greater than 5, the conditions for the chi-square goodness-of-fit test are present.

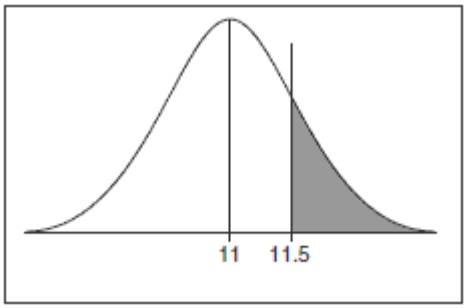
$$X^2 = \frac{(12-15.5)^2}{15.5} + \frac{(19-15.5)^2}{15.5} + \cdots + \frac{(26-15.5)^2}{15.5} + \frac{(5-15.5)^2}{15.5} = 31.14$$

df = (12–1)(2–1) = 11 $\Rightarrow$ 0.001 < *P*-value < 0.0025 (from Table C; using the TI-83/84, we have X^2cdf (31.14,1000,11)= 0.00105).
Because the *P*-value is so small, we have evidence to reject the null and conclude that birth month and draft number are not independent. That is, month of birth is related to draft numbers. Observation of the data indicates that the lottery was biased against people born later in the year. That is, people born later in the year were more likely to get drafted.

Solutions to Practice Exam 1, Section II, Part B

Solution to #6

1. The situation described is pictured below.



Let X be the length of a fish. Then $P(X > 11.5) = P\left(z > \frac{11.5 - 11}{0.6} = 0.83\right)$ =1– 0.7967 = 0.2033 (from Table A; using the TI-83/84, we have $P(X > 11.5)$ = normalcdf (0.83, 100) or normalcdf (11.5, 1000, 11, 0.6)). Rounding to the nearest tenth, $P(X > 11.5) = 0.2$.

2. Because the probability that the fish is large enough to keep is 0.2, let the digits 0 and 1 represent a fish that is large enough to keep and the digits 2, 3, 4, 5, 6, 7, 8, and 9 represent fish that must be released. Begin at the first line of the table and count the number of digits required before five of the digits 0, 1 are found.

3. On the random number table below, successes (a large enough fish) are in boldface. Backslashes separate the various trials. The number of catches it took to get two sufficiently large fish is indicated under each separate trial.

79692	51**7**0**7** (26 fish)	73274	**1**2548	9**1**497	**1****1**1**3**5	8**1**2**1**8 (15 fish)	79572	**0**\\6484	87**44**0
4**1**957	2**1**6**07** (21 fish)	5**1**\\248	54772	**1**9**4**8**1**	9**0**392	35268 (34 fish)	36234	9**0**244	**0**\\2146
07094 (14 fish)	3**1**75**0**\\	69426	62510	90127	43365	61167	53938	03694	76923
59365	43671	12704	87941	51620	45102	22785	07729	40985	92589

4. Based on the five trials, an estimate of the average number of trials required to get two fish of minimum size is (26 + 15 + 21 + 34 + 14)/5 = 22.

5. The expected wait to catch a single fish with $P = 0.2$ is 1/0.2 = 5 fish. The expected wait to catch five fish is then 5(5) = 25.