AP Calculus AB Chapter 4 Review – Skills 2011 **Name** _____

1. Evaluate the following definite and indefinite integrals:

a. $\int_{-1}^{2} |2x-1| \, dx$

b. $\int x\sqrt{3x+1} \, dx$

c. $\int \cos(3x)\left[\sin(3x)-1\right]^4 \, dx$

d. $\int \frac{(15x^2+10)}{(x^3+2x-3)^4} \, dx$

e. $\int_{1}^{4} \frac{1}{\sqrt{x}\left(\sqrt{x}+1\right)^3} \, dx$

f. $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(x^3+\sin x\right) dx$

2. Let R be the region in the first quadrant under the graph $y = \dfrac{x}{\left(x^2+2\right)^2}$ for $0 \le x \le \sqrt{6}$.

a. Find the area of R.

b. If the line $x = k$ divides R into 2 equal regions, find k.

3. If $F(x) = \int_2^{4x} 2^{(t+1)}\,dt$, write the equation of the line ***normal*** to $F(x)$ at $x = \dfrac{1}{2}$.

4. Convert the Riemann Sum to an integral: $\lim_{n\to\infty}\left(\frac{1}{n}\right)\left(\sqrt{\cos\left(\frac{n}{n}\right)^2} + \sqrt{\cos\left(\frac{n+1}{n}\right)^2} + \ldots + \sqrt{\cos\left(\frac{10n}{n}\right)^2}\right)$

5. Let f be a function such that $f''(x) = 6x + 8$

a) Find $f(x)$ if the graph of f is tangent to the line 3x - y = 2 at the point (0, -2).

b) Find the *average* value of $f(x)$ on the closed interval [-1, 1}.

6. $f(x)$ is an ***odd*** function and g(x) is an ***even*** function.

$\int_{-3}^{3} g(x)dx = 10$, $\int_{5}^{3} g(x)dx = -4$, $\int_{2}^{5} f(x)dx = 7$, $\int_{-2}^{0} f(x)dx = -3$, $\int_{0}^{7} f(x)dx = -6$

Find: a) $\int_{0}^{3} g(x)dx =$

b) $\int_{0}^{5} g(x)dx =$

c) $\int_{-5}^{0} 2f(x) + 6\ dx$

d) $\int_{5}^{7} f(x)dx - \int_{3}^{5} g(x)dx$

7. A particle moves on the *x*-axis so that its velocity (m/sec) at any time t ≥ 0 is given by $v(t) = t^3 - 8t^2 + 15t - 2$. At $t = 0$, the particle is 4 meters to the left of the origin.
Calculator allowed.

a) Set up an integral and find the particle's change in position over the first 4 seconds.

b) Set up an integral and find the position of the particle at $t = 5$ seconds.

c) Set up an integral and find the total distance traveled by the particle from $t = 2$ to $t = 5$ seconds.

d) Determine the velocity when the acceleration is $3.5\ m/\sec^2$ in the first 3 seconds.
Show the mathematical setups.

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8. A radio-controlled experimental vehicle is tested on a straight track. It starts from rest, and its velocity $v(t)$ (meters per second) is recorded in the table every 10 seconds for 80 seconds. Approximate the total distance traveled by the vehicle using the following methods: ***Show set up and solve.***

Time (seconds)	0	10	20	30	40	60	80
V(t) (m/sec)	0	5	21	40	62	78	83

a) Left or Right Sum, whichever one produces an overestimation.

b) Midpoint Sum

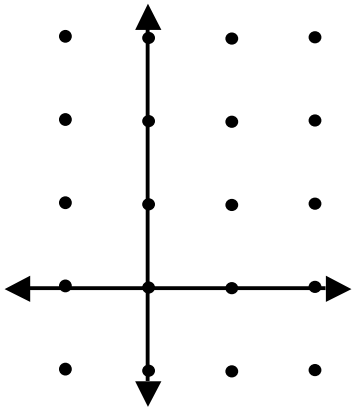
c) Trapezoidal Sum

9. Given $f(x) = \cos x + \frac{x}{2} - 3$, find the absolute maximum of *f(x)* on the interval [0, 2π].

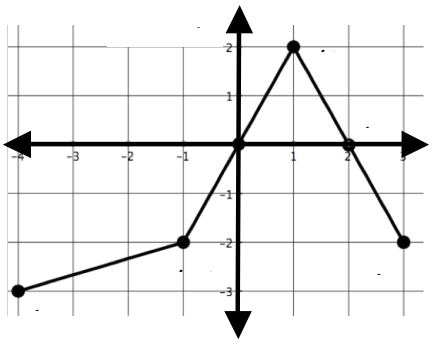
Show mathematical setup. Justify your answer using Calculus.

10. Sketch the slope field given $\frac{dy}{dx} = y + 1$.

Draw the particular solution passing through $(0,0)$.



11. The graph of the function f at the right consists of three line segments.



a) Let g be the function given by $g(x)=\int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$ *and* $g''(-1)$, find the value or state that it does not exist.

b) For the function g defined in *part a*, find the x-coordinate of each point of inflection of the graph of g on the open interval $-4<x<3$. *Explain your reasoning.*

c) Let h be the function given by $h(x)=\int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4\le x\le 3$ for which $h(x)=0$.

d) For the function h, defined in *part c*, find all intervals on which h is decreasing. *Explain your reasoning.*

e) Let $k(x)=\int_{0}^{2x} f(t)\,dt$, find $k(-2)$ *and* $k'\left(\frac{1}{2}\right)$.

12. At 8:00 pm, there were 1,350 students at the Winterfest Dance. Students continued to enter the dance at a rate which is described by the equation $E(t)$. Students were leaving the dance to get refreshments in the commons at a rate which is described by the equation $L(t)$.
Let $t = 0$ is 8:00 pm.

a. Set up an integral $N(t)$ to determine how many students were at the dance at ll:00 pm.

b. Set up an integral $N(t)$ to determine how many students were at the dance at any time t.

c. Find $N'(t)$ and explain what this means.

Answers:

1. a. $\frac{9}{2}$ b. $\frac{2}{45}(3x+1)^{\frac{5}{2}} - \frac{2}{27}(3x+1)^{\frac{3}{2}} + C$ c. $\frac{1}{15}[\sin\ (3x\text{-}1)]^5 + C$ d. $\frac{-5}{3(x^3+2x-3)^3} + C$ e. $\frac{5}{36}$ f. 0

2. a. $\frac{3}{16}$ b. $K = \frac{\sqrt{30}}{5}$ 3. $\left(\frac{1}{2},0\right)$, $m_{\tan} = 2^5$, $m_{norm} = -\frac{1}{32}$, $y = -\frac{1}{32}\left(x - \frac{1}{2}\right)$ 4. $\int_1^{10} \sqrt{\cos x^2}\ dx$

5. a. $f(x) = x^3 + 4x^2 + 3x - 2$ b) $\frac{-2}{3}$ 6. a. 5 b. 9 c. 10 d. -20

7. a. 5.333 m b. -3.583 m c. 11. 066 m d. $v(.856203) = 5.606\ m/\sec$

8. a. $A_R \approx 4500$ meters b. $A_M \approx 4020$ meters c. $A_T \approx 3980$ meters 9. $\pi - 2$ is the absolute maximum 10. Slope Field

11. a) $g(-1) = \frac{15}{2}$, $g'(-1) = -2$, $g''(-1)$ does not exist because f is not differentiable at $x = -1$.
b) $x = 1$ $g''(1)$ does not exist and $g' = f$ changes from increasing to decreasing at $x = 1$.
c) $x = -1, 1, 3$
d) h is decreasing on $(0, 2)$ *because* $h'(x) = -f(x) < 0$ *or* $f(x) > 0$.
e) $k(-2) = \frac{17}{2}$, $k'\left(\frac{1}{2}\right) = 4$

12. a. $1350 + \int_0^3 E(t) - L(t)\,dt$ b. $1350 + \int_0^t E(x) - L(x)\,dx$
c. $N'(t) = E(t) - L(t)$ which means the rate of change of the number of students at the dance at any time t.