AP Calculus AB 2010
Chapter 4 Review – *Conceptual* **Name**_____

Evaluate the following indefinite integrals:

1. $\int (1-x)\sqrt{x}\ dx$

2. $\int \frac{(a-9)(a+9)}{a^2}\ da$

3. $\int \frac{\csc^3\theta + 2\cot\theta}{\csc\theta}\ d\theta$

4. Find the particular solution of the differential equation $f''(x) = x(3x+2)$ whose graph passes through the point $(-2,4)$ and at that point is tangent to $3x + y = 4$.

5. Convert the following Reimann Sums into a definite integral.

a. $\lim_{n\to\infty} \frac{1}{n}\left[\sec^2\left(\frac{4n}{n}\right) + \sec^2\left(\frac{4n+1}{n}\right) + \sec^2\left(\frac{4n+2}{n}\right) + \ldots + \sec^2\left(\frac{9n}{n}\right)\right]$

b. $\lim_{n\to\infty} \frac{1}{n}\left[5\sqrt{\left(\frac{3}{n}\right)+4} + 5\sqrt{\left(\frac{3}{n}+\frac{1}{n}\right)+4} + 5\sqrt{\left(\frac{3}{n}+\frac{2}{n}\right)+4} \ldots + 5\sqrt{\left(\frac{10n}{n}\right)+4}\right]$

6. *Water Pollution* – Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased leakage each hour, recorded in the table below.

Time (hrs.)	0	8	16	30	44	52	60
Leakage (gal./hr.)	50	70	97	136	190	265	369

Approximate the total quantity of oil that has escaped after 60 hours. Show set up.

a. Find the approximation that will produce an overestimate of the total amount of leakage. Explain why you chose the method that you used.

b. If the tanker originally contained 2.5 million gallons of oil, give the most accurate estimate for the amount of oil remaining in the tanker after 60 hours. Show all work.

7. Evaluate the following definite integrals.

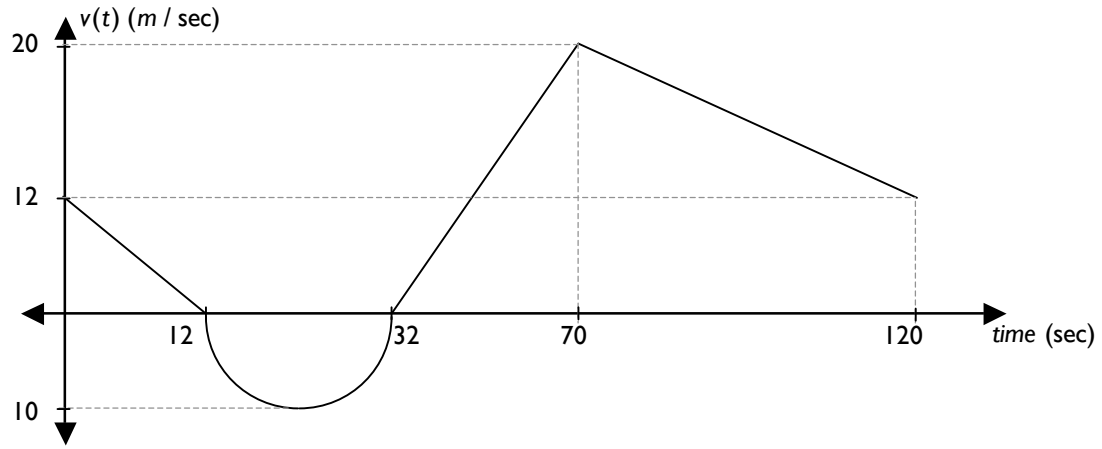
a. $\int_{-1}^{1} x^3 + x - 1 \, dx$

b. $\int_{0}^{\frac{\pi}{3}} \sec^2(x) \, dx$

c. $\int_{-1}^{0} (x+2)(x^2+4x+3)^5 \, dx$

d. $\int_{0}^{\frac{\pi}{4}} \sec^4 x \tan x \, dx$

8. Given the graph of velocity, $v(t)$, where velocity is measured in meters/sec, answer the following.



a. What is the total distance traveled over the 2 minutes? *Show your work.*

b. When is the particle speeding up? *Justify your answer.*

c. When is the particle farthest to the left? *Show your work.*

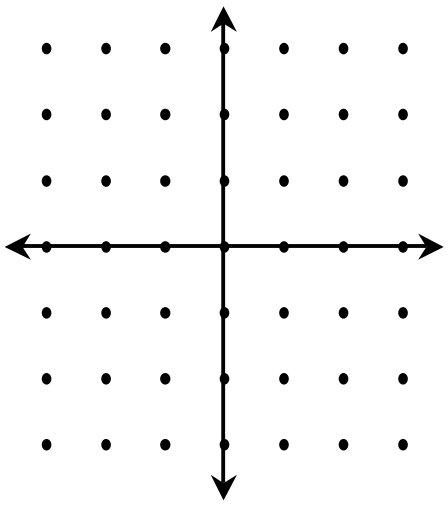
d. Where is the particle, with respect to its starting place, at $t = 70$ seconds? *Show your work.*

e. Find the speed of the particle at $t = 33$ seconds.

9. Complete the slope field on the graph for $-2 \le x \le 3$ and $-2 \le y \le 3$ on the graph.

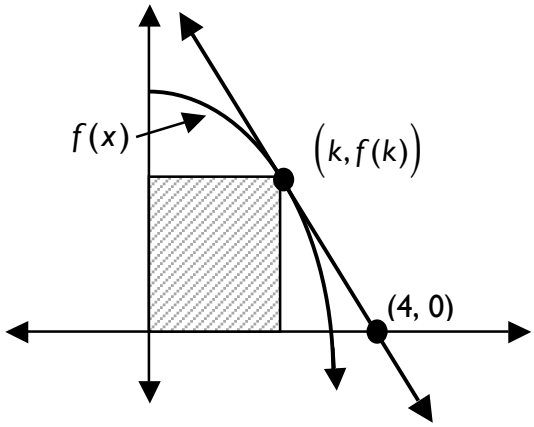
a. $\dfrac{dy}{dx} = -\dfrac{1}{x} + x$

b. Sketch a curve for $f(x)$ on the graph if $f(1) = 1$.

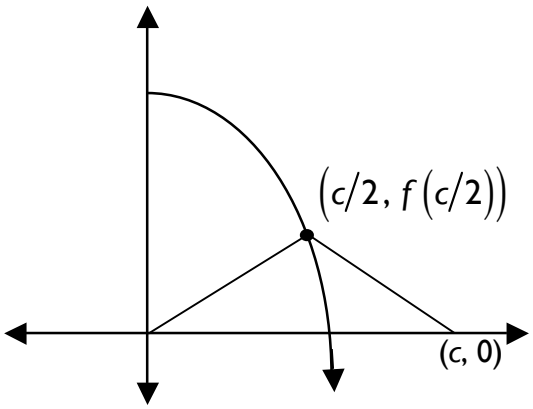


10. Let $f(x)=12-x^2$ for $x \geq 0$ and $f(x) \geq 0$.

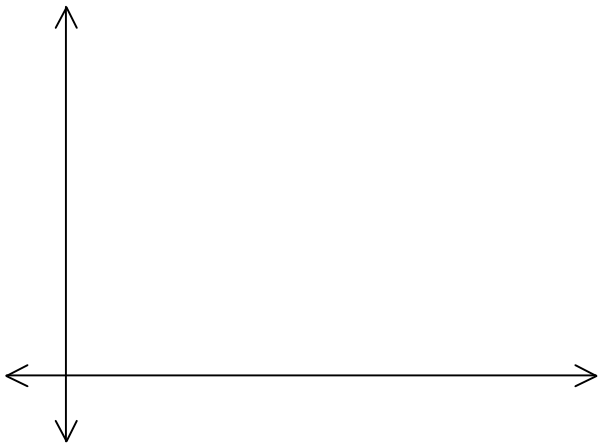
a. The line tangent to the graph of $f(x)$ at the point $(k, f(k))$ intercepts the *x*-axis at *x* = 4. What is the value of *k* if the point of tangency is in quadrant I?



b. An isosceles triangle whose base is the segment from $(0,0)$ to $(c,0)$ has its vertex on the graph of $f(x)$. For what value of *c* does the triangle have a maximum area? ***Justify your answer.***



11. Given $f''(x)=6x$, where $f'(0)=1$ and $f(-2)=-6$, find the sum which produces an overestimate for the area under the curve of $f(x)$ from $x=0$ to $x=4$ with 4 partitions. ***Show set-up.***



12. Determine the ***x*** values of the absolute maximum and minimum in the closed interval $[-2,3]$ if $f(x) = 2x^3 + 3x^2 - 12x$. ***Justify your answer.***

13. Given $f'(x) = x\cos(2x^2 + 1)$, find $f(x)$.

14. Given a function $f(x)$ and $f(0) = 2$, where $f'(x) = \cos x - \frac{1}{2}x$ over the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Determine the point(s) on $f(x)$ where a point of inflection exists.

State your answer correct to three decimal places.

Answers

1. $\frac{2}{3}x^{3/2}-\frac{2}{5}x^{5/2}+C$ 2. $a+\frac{81}{a}+C$ 3. $-\cot\theta+2\sin\theta+C$ 4. $f(x)=\frac{x^4}{4}+\frac{x^3}{3}+x+\frac{14}{3}$

5. a. $\int_4^9 \sec^2 x\,dx$ b. $\int_0^{10} 5\sqrt{x+4}\,dx$

6. a. Right Sums: Total Quantity of Oil = 10972 gallons
   b. Trapezoidal Sum: The amount still in the tanker is 2,490,583 gallons.

7. a. -2 b. $\sqrt{3}$ c. $\frac{243}{4}$ d. 0.75

8. a. (1252 + 50π) meters b. $(12,22)\cup(32,70)$ c. 32 seconds d. (452 – 50π) meters to the right
   e. $\frac{10}{19}$ *m* / sec

9. Check with others or teacher

10. a. $k = 2$ b. $c = 4$

11. $f(x)=x^3+x+4$ $A_R \approx 126$ sq units

12. $x = 1$ abs. min; $x = 3$ abs. max

13. $f(x)=\frac{1}{4}\sin(2x^2+1)+C$

14. $(-0.524, 1.431)$