AB Calculus Chapter 7 Review 2011 **Name** _____

1. *Evaluate the following integrals. State the strategy used.*

a. $\int 4x^2 \tan(x^3 - 3)\, dx$ _____

b. $\int \frac{6t^2 + 5t + 14}{2t + 3}\, dt$ _____

c. $\int_{e^2}^{e^3} \frac{(\ln x)^3}{x}\, dx$ _____

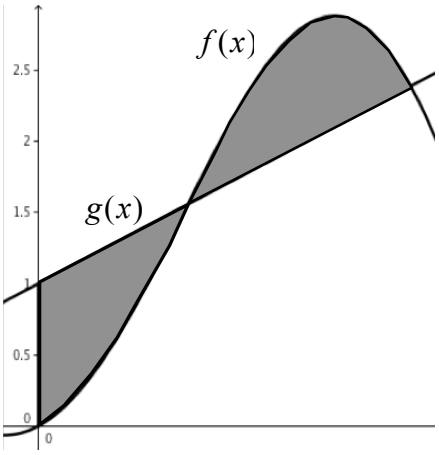
d. $\int x(3x+1)^3 dx$ _____

e. $\int x^2 \sqrt{1 + \tan^2(x^3)}\, dx$ _____

f. $\int 4x\ e^{6x^2+2} dx$ _____

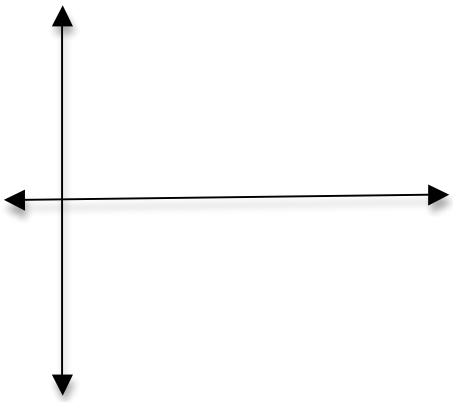
2. Find the total area between $f(x) = x\sin x + \frac{x}{2}$ and $g(x) = \frac{1}{2}x + 1$, between $x = 0$ and $x = 2.773$.

Set up integral and evaluate ***<u>with</u>*** *calculator.*



3. Find the volume of the solid that results when the area of the region enclosed by $y = x^2$, $y = 0$, and $x = 2$ is revolved about the following: *Set up integrals and evaluate* ***<u>without</u>*** *a calculator!*

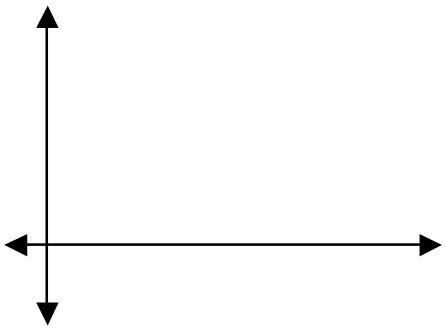
a. $y = -3$



b. $x = 2$

4. A region, $\boldsymbol{R}$, in the first quadrant is bounded by $y = e^x$ and $y = 2x + 1$.

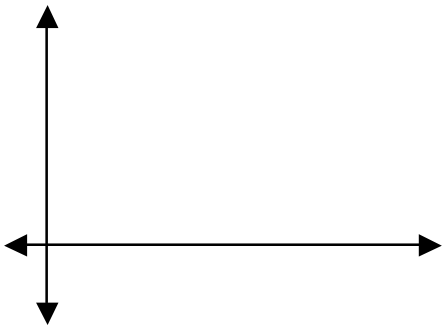
a. Find the volume of the solid when the region, $\boldsymbol{R}$, is rotated about the x – axis. *Set up the integral. DO NOT SOLVE.*



b. Find the volume of the solid when the region, $\boldsymbol{R}$, is rotated about the y – axis. *Set up the integral. DO NOT SOLVE.*

c. Find the volume of the solid whose base is region, $\boldsymbol{R}$, and has quarter circle cross sections that are perpendicular to the x - axis. *Set up the integral. DO NOT SOLVE.*

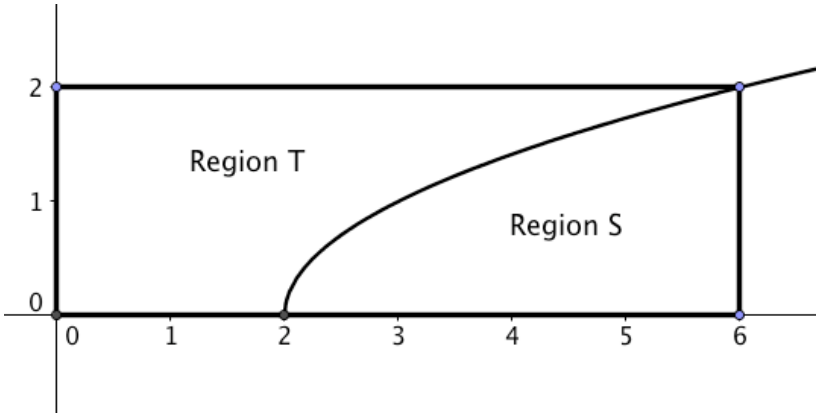
d. Find the volume of the solid whose base is region, $\boldsymbol{R}$, and has isosceles right triangles with a leg in the base perpendicular to the y – axis. *Set up the integral. DO NOT SOLVE.*



5. The two regions, T and S, are bounded by $y = \sqrt{x-2}$, $y = 0$, $y = 2$, and $x = 6$, as shown in the diagram.

a. If region S is rotated about the x – axis to create a solid with a volume of 8π cubic units is formed.

A line $x = k$ is drawn through the region so that it divides the solid created into two parts having equal volumes. Find the value of k that will make this happen.
Calculator Needed in last step to find *k*.



b. The region T, shown in the diagram, is rotated about the x–axis to create a solid. Determine the volume of the solid. **Write an integral expression and solve using the calculator.**

6. Write the equation of the tangent to $y = x^{\sin x}$ at $x = \pi$.

7. Find the area between the curve $x + y = 2$ and $x = y^2 - 4$. *Set up integral and evaluate* ***with*** *a calculator*.

8. A function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x, y) on the graph is given by $\frac{dy}{dx} = y^2(6 - 2x)$.

(a.) Find $\frac{d^2 y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

(b.) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6 - 2x)$ with the intial condition $f(3) = \frac{1}{4}$.

Answers:

1. a. $\frac{-4}{3}\ln\left|\cos(x^3-3)\right|+C$ *or* $\frac{4}{3}\ln\left|\sec(x^3-3)\right|+C$

   b. $\frac{3}{2}t^2-2t+10\ln\left|2t+3\right|+C$

   c. $\frac{65}{4}$

   d. $\frac{(3x+1)^5}{45}-\frac{(3x+1)^4}{36}+C$

   e. $\frac{1}{3}\ln\left|\sec x^3+\tan x^3\right|+C$

   f. $\frac{1}{3}e^{6x^2+2}+C$

2. 1.590 sq. units

3. a. $\frac{112\pi}{5}$ cu. units b. $\frac{8\pi}{3}$ *units*^{3}

4. a. $V=\pi\int_0^{1.25643}(2x+1)^2-(e^x)^2\ dx$ b. $V=\pi\int_1^{3.51286}(\ln\left|y\right|)^2-(.5(y-1))^2\ dy$

   c. $V=\frac{\pi}{4}\int_0^{1.25643}(2x+1-e^x)^2\ dx$ d. $V=\frac{1}{2}\int_0^{1.25643}(2x+1-e^x)^2\ dx$

5*a*. $k=4.828$ 5*b*. 16π Hint: You need two integrals, 0 to 2 and then 2 to 6.

6. $y-1=-\ln\pi(x-\pi)$ 7. 20.833 sq. units

8 .a. $\frac{d^2y}{dx^2}=2y^3(6-2x)^2-2y^2$, $\left.\frac{d^2y}{dx^2}\right|_{(3,\frac{1}{4})}=-\frac{1}{8}$ 8b. $y=\frac{1}{x^2-6x+13}$