AB Calculus Review 2010 Name_____

Chapter 2 - Part 2 - *Skills* Team # _____ Period _____

*The **ART OF REWRITING TECHNIQUES** will lead to a more efficient and elegant solution.*

Show all work! No Calculator except for simple calculations unless otherwise stated!

1. Find $f'(x)$ if $f(x)=\frac{1}{5x^2}+\frac{2}{\sqrt[3]{x}}+\pi$.

2. $\frac{d}{d\theta}\left[\sec\theta(1+\cot\theta)\right]$

3. Find y' if $y=\frac{\csc(2x)}{x^2-3}$.

4. Find the slope of the tangent line of

$f(x)=x\sqrt[3]{x^2+1}$.

5. Find the instantaneous rate of change of

$x(t)=3t\left[\sec^2(6t)-\tan^2(6t)\right]\cot t$.

6. Find $a(t)$ if $v(t)=4\sin^2(2t)-3\tan(2t)$.

7. Evaluate: $\lim_{h \to 0} \dfrac{\csc 3x - \csc 3(x-h)}{h}$

8. Given that the formula for the area of an equilateral triangle is $A = \dfrac{s^2\sqrt{3}}{4}$ and its perimeter is $P = 3s$. Find the rate of change of the area with respect to the perimeter and then find the rate of change of the area at the instant the perimeter is 10 cm.

9. Use the ***Formal Definition of Differentiability*** to find (a, b) if $f(x)$ is differentiable at $x = 3$,

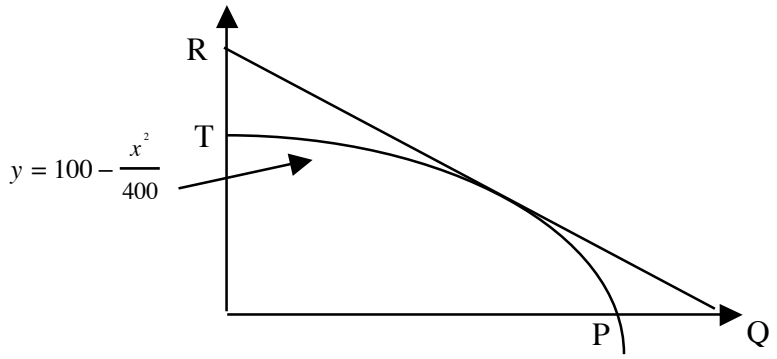
given $f(x) = \begin{cases} 4x+3, & x \le 3 \\ ax^2 - bx, & x > 3 \end{cases}$.

$f'(x) = \{$

10. Given $y = \dfrac{2x}{x+1}$ and $t = x^2 + 4x$. If y = *f(t)* is graphed in the $t - y$ plane, write the equation of the **normal** line to y at $t = -3$.

11. Suppose that the graph of $y = 100 - \dfrac{x^2}{400}$ represents a hill rising from level ground represented by the x-axis as shown in the figure. Let x and y be measured in feet. A 16 foot tower, $\overline{TR}$ stands on the top of the hill.

a. Determine an equation of the line of sight from the top of the tower tangent to the hill.



b. From the top of the tower, the ground between P and Q is blocked by the hill and cannot be seen. Find the distance between P and Q to the nearest foot.

12. a. Given $x^2 - xy + y^2 = 9$, find $\frac{dy}{dx}$.

b. Find the points on the curve where the tangents are vertical.

c. Find the equation(s) of the horizontal tangents.

d. At $(0,3)$, find the ***rate of change*** of the ***slope***.

13. Given $f(x) = \cos(x)\sin(x)$.

a) Find the equation of the tangent line to $f(x)$ at $x = \frac{\pi}{3}$. Do <u>not</u> use a calculator!

b) Use the equation of the tangent line to approximate the value of $f(1)$. Round to the nearest thousandth.

14. The distance in meters, a dragster covers in t seconds on a straight road is modeled by the function

$d(t) = 2t^3 + t^2 + 3t$.

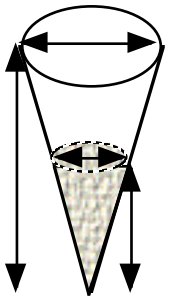
a). Find the average velocity in the first 4 seconds.

b). Find when in the first 4 seconds that the dragster's instantaneous velocity was at this average velocity. Calculator allowed!

c). Find the dragster's average acceleration over the first 4 seconds.

Review/ Preview:

15. Given a conical tank with a diameter of 6 meters and a height of 10 meters. If water is poured into the tank as shown, write an equation of the volume of the water as a function of the depth of the water. (Do not solve.)



Mean Value Theorem, Rolles Theorem and L'Hopitals Rule…

ANSWERS:

1. $f'(x) = \frac{-2}{5x^3} - \frac{2}{3\sqrt[3]{x^4}}$

2. $\sec\theta\tan\theta - \csc\theta\cot\theta$

3. $y' = \frac{-2x^2\csc(2x)\cot(2x) + 6\csc(2x)\cot(2x) - 2x\csc(2x)}{(x^2-3)^2}$

4. $f'(x) = \frac{\left(\frac{5x^2}{3}+1\right)}{(x^2+1)^{\frac{2}{3}}}$

5. $3\cot t - 3t\csc^2 t$

6. $a(t) = 16\sin(2t)\cos(2t) - 6\sec^2(2t)$ - Can you figure out why this is equivalent to: $a(t) = 8\sin(4t) - 6\sec^2(2t)$?

7. $-3\csc(3x)\cot(3x)$

8. $\frac{dA}{dP} = \frac{\sqrt{3}}{6}s, \frac{dA}{dP}\Big|_{P=10} = \frac{5\sqrt{3}}{9}cm$

9. $a = \frac{-1}{3}, b = -6$

10. $y - 3 = 4(t+3)$

11. $a.\ y - 84 = \frac{-2}{5}(x-80)$ $b.\ 90\ feet$

12. $a.\ \frac{dy}{dx} = \frac{y-2x}{2y-x}$ b. $(2\sqrt{3},\sqrt{3}),(-2\sqrt{3},-\sqrt{3})$

    c. $y = 2\sqrt{3}, y = -2\sqrt{3}$ d. $\frac{-1}{4}$

13. a) $y - \frac{\sqrt{3}}{4} = -\frac{1}{2}\left(x - \frac{\pi}{3}\right)$ b) 0.457

14. a) 39 m/s b) 2.288 s c) 26 m/s^2

15. $V(h) = 3\pi h^3/100$