AP Calculus AB
Chapter 2 - Part One

Review Sheet 2010

Name: _____
Due: Day of Test 9/24/10

Learning Targets: Students will be able.

1. To sketch an approximate graph of the derivative given the graph of a function, and the reverse.
2. To use the limit definition(s) of the derivative – RDQ, and LDQ to determine the derivative of a function.
3. To apply the Numeric Difference Quotient to determine the value of the derivative at a point.
4. To write the equation of a tangent line and a normal line.
5. To determine if a function is differentiable at a point, both graphically and numerically.

1. Find the derivative of functions.

a.) $f(x) = \sqrt{x+1}$ use RDQ

b.) $f(x) = \dfrac{-3}{x-4}$ use LDQ

c.) $f(x) = 2x^2 - x$ use RDQ

d.) Given a function $f(x)$ which has $f'(x) = \dfrac{2x+1}{x^2-9}$, find the x –values for any horizontal or vertical tangent lines.

2. For the following, identify the type of difference quotient and determine the function $f(x)$.

a. $f'(x) = \lim_{h \to 0} \dfrac{2x^3 - 5x - \left[2(x-h)^3 - 5(x-h)\right]}{2h}$

b. $f'(x) = \lim_{x \to \frac{2\pi}{3}} \dfrac{4\cos x + 2}{x - \frac{2\pi}{3}}$

c. $f'(x) = \lim_{h \to 0} \dfrac{\frac{2}{\sqrt{x+h+5}} - \frac{2}{\sqrt{x+5}}}{h}$

d. $f'(x) = \lim_{h \to 0} \dfrac{\frac{\sin 5x}{3x} - \frac{\sin 5(x-h)}{3(x-h)}}{h}$

3. Use the numerical difference quotient to find the derivative of the function at a point.

a. Find $f'(2)$ given $f(x) = x^2 - 1$.

b. Given: $f(x) = \sqrt{2x+3}$. Find $f'(1)$.

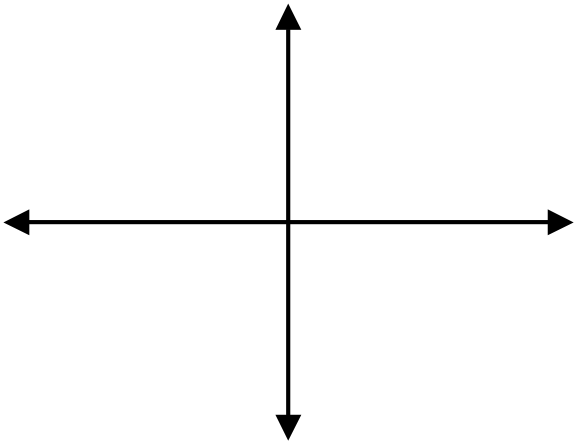
4. Draw the graph of a function $f(x)$ with the following characteristics. (Label your axis)

$f'(x) > 0 \;\; for \;\; x < -3$

$f'(x) < 0 \;\; for \;\; -3 < x < 7$

$f'(x) > 0 \;\; for \;\; x > 7$

$f'(-3) = 0 \;\; and \;\; f'(7) \; DNE$



5. **Using your calculator**, graph the function $f(x) = e^x - \frac{3}{x}$. Show the mathematical set-up.

a. State the domain.

b. State the range.

c. Find the zero(s) of $f(x)$.

d. Find $f(0)$.

e. Find the average rate of change in the interval $[1, 4]$.

f. Find the instantaneous rate of change at $x = 4$.

g. Write the equation of the tangent line to $f(x)$ at $x = 4$.

h. Write the equation of the normal line to $f(x)$ at $x = 4$.

i. Graph the tangent line in Y_2 and check if this line is tangent for $f(x)$ at $x = 4$.

6. Given a function $f(x)$ which is continuous for all values of x and $f'(4) = -7$.

a. What is happening to $f(x)$ when x=4 based on the derivative?

b. If $f(x)$ is an even function, what is the value of $f'(-4)$? Explain your answer.

c. If $f(x)$ is an odd function, what is the value of $f'(-4)$? Explain your answer.

7. **By hand**, given the curve $f(x) = 25 - x^2$:

a. Calculate the slope of the secant line between the y-intercept and the x-intercept on the positive side of the x-axis.

b. Locate the point on the curve where the slope of the tangent equals the slope found in part 5a if $f'(x) = -2x$.

c. Write the equation of the line through the point of tangency found in part 5b.

8. Given the table, you may assume *g(x)* is continuous and monotonic between the given values of *x*. Estimate the following:

a.) $g(-2) =$ _____ b.) $\lim_{x \to -2^-} g'(x) \approx$ _____

c.) $\lim_{x \to -2^+} g'(x) \approx$ _____

d.) Is *g(x)* differentiable at $x = -2$? Justify your answer using the formal definition of differentiability.

x	$g(x)$
-2.003	3.465
-2.002	3.780
-2.001	3.889
-2	4
-1.999	3.910
-1.998	3.750
-1.997	3.413

9. Given the table of values for x, $f(x)$, and $f'(x)$, you may assume *f(x)* is continuous and monotonic between all the given values of x.
Answer the following:

a.) $\lim_{x \to 3^-} f(x) =$ _____

b.) $\lim_{x \to 3^+} f(x) =$ _____

c.) $\lim_{x \to 3^-} f'(x) =$ _____

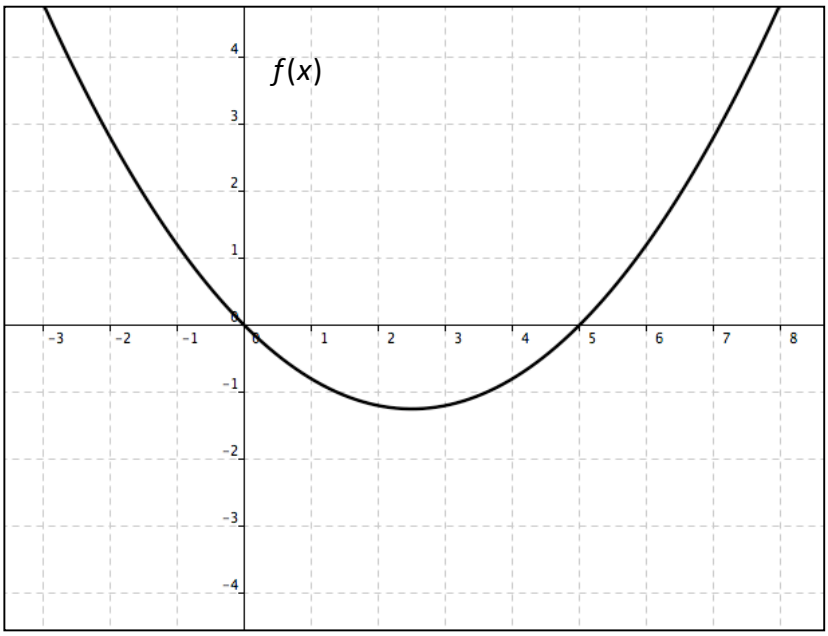
d.) $\lim_{x \to 3^+} f'(x) =$ _____

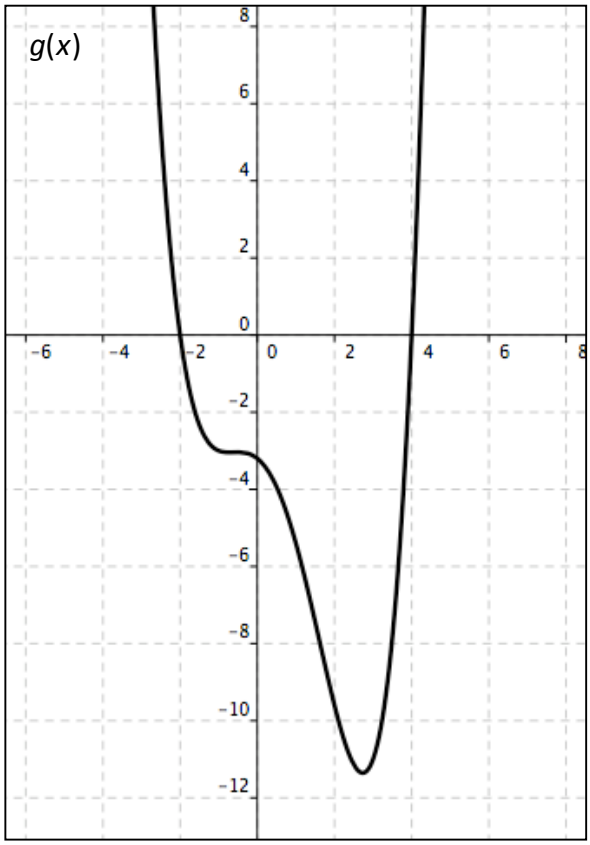
e.) $f'(3) =$ _____

f.) Is $f(x)$ differentiable at $x = 3$? Explain your answer in complete sentences.

x	$f(x)$	$f'(x)$
2.997	7.9988	.0016
2.998	7.9996	.0009
2.999	7.9998	.0003
3	8	DNE
3.001	8.9979	-.0006
3.002	8.9765	-.0015
3.003	8.9750	-.0019
3.004	8.9748	-.0020

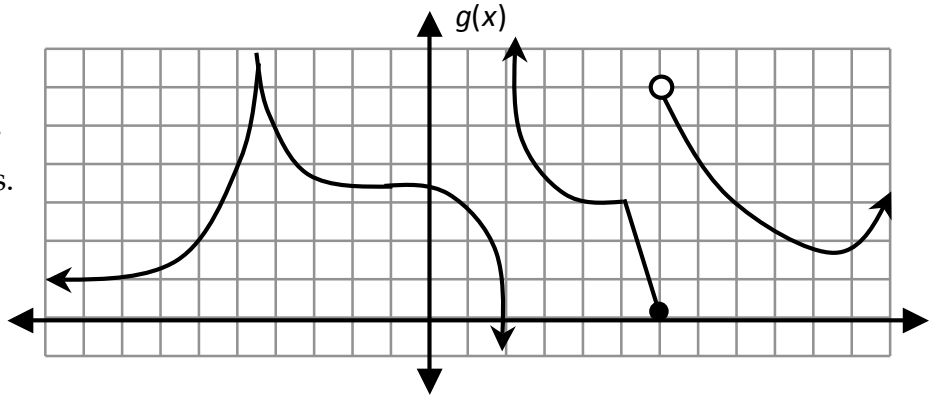
10. For the functions *f* and *g* shown in the graphs, draw the graphs of their derivatives on the same axis.





11. Review-Preview: Factor $8(2x+1)^{\frac{2}{3}} + (2x+1)^{-\frac{1}{3}}(x-3)$

12. Determine the x-coordinates of all locations where the function $g(x)$ is not differentiable. State what part of the Definition of Differentiability fails.



Answers

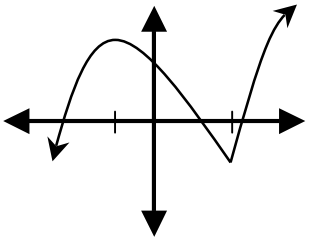
1a. $f'(x) = \dfrac{1}{2\sqrt{x+1}}$ 1b. $f'(x) = \dfrac{3}{(x-4)^2}$ 1c. $f'(x) = 4x - 1$ 1d. $HA\ at\ x = \dfrac{1}{2}$; $VA\ at\ x = 3, x = -3$

2. a. $LDQ\quad f(x) = 2x^3 - 5x$ b. $NDQ\quad at\ x = \dfrac{2\pi}{3}\quad f(x) = 4\cos x$

c. $RDQ\quad f(x) == \dfrac{2}{\sqrt{x+5}}$ d. $LDQ\quad f(x) = \dfrac{\sin 5x}{3x}$

3 a. $f'(2) = 4$ b. $\dfrac{1}{\sqrt{5}}$

4. One possible sketch



5. a. $D: (-\infty, 0) \cup (0, \infty)$ b. $R: (-\infty, \infty)$ c. $x = 1.05$ d. undefined

e. 18.043 f. $m = 54.786$ g. $y - 53.848 = 54.786\,(x-4)$ h. $y - 53.848 = -.018\,(x-4)$

6. a. $f(x)$ is decreasing

b. 7, if the function is reflected over the y-axis, the slope will change sign.

c. - 7 because the function is reflected over the origin, the slope should not change.

7. a. $m = -5$ b. $\left(\dfrac{5}{2}, \dfrac{75}{4}\right)$ c. $y - \dfrac{75}{4} = -5\left(x - \dfrac{5}{2}\right)$

8. a. 4 b. 111 c. - 90 d. not diff. - as the limit DNE

9. a. 8 b. 9 c. 0 d. 0 e. DNE f. No

10. $f'(x)$ should be a linear with an x-intercept at $x = 2.5$ and a positive slope

$g'(x)$ should be a cubic with an even x-intercept at $x = -1$ and an odd x-intercept at $x = 3$.

11. $(2x+1)^{-\frac{1}{3}}(17x+5)$

12. *fails the def. of diff. at* $x = -4.5$ *and* 4, *not cont. at* $x = 2, 5$ *and* 6.