



Implicit Differentiation

Example: Given $4x^2 - xy - y^2 = 1$ find $\frac{dy}{dx}$.

- derivative of y^n is $ny^{n-1} \frac{dy}{dx}$
- watch for product rule and distribution of negative signs
- horizontal tangents occur where the numerator equals zero
- vertical tangents occur where the denominator equals zero
- after finding critical x or y coordinates, sub back into the original function to find missing coordinates
- when finding critical points, **check to ensure the points are on the curve**
- second derivatives will include the first derivative as part of the function

Related Rates

Example(s): Ladder problems, Pythagorean problems, similar triangles, similar cones.

- draw a diagram and label with variables and constants.
- Determine the “when” statement.
- Write an equation that relates the variables.
- Take derivative with respect, usually with respect to time.
- Sub in “when” statement values and solve.

Velocity/Acceleration

- Acceleration is the derivative of the velocity which is the derivative of position.
- Sign of velocity determines the direction of the motion.
- Vel./Accel. have same sign – speeding up, have opposite signs – slowing down.
- Speed increasing means speeding up - Speed decreasing means slowing down.

- $\int_a^b v(t) dt =$ net change in position over the interval (displacement)

- $\int_a^b |v(t)| dt =$ total distance traveled over the interval

- $\int_a^b a(t) dt =$ net change in velocity over the interval

- speed is the absolute value of velocity $\text{speed} = |v(t)|$

- average velocity $= \frac{s(b) - s(a)}{b - a}$ or $\frac{1}{b - a} \int_a^b v(t) dt$

- $s(b) = s(a) + \int_a^b v(t) dt$ calculates the position at time $t = b$.

Area Approximations

- Area under a curve can be approximated using Right Sums, Left Sums, Midpoint Sums, (area of rectangles) and/or the Trapezoidal Rule (area of trapezoids).
- A Riemann Sum means use rectangles to find area.
- From a function with an interval, you can calculate a constant value for delta x.
- Right, Left, and Midpoint sums have “n” terms and Trapezoidal Rule has “n+1” terms.
- From a table where delta x is not constant, you will have to calculate the area of each individual partition, whether it is a rectangle or a trapezoid.